# PRABHU LAL BHATNAGAR

## (1912—1976)

### Elected Fellow 1950

"That all the world may say, 'This was a man' ". These words could have been written of PRABHU LAL BHATNAGAR. There were only a few, who came in contact with him and went away unimpressed. Here was a man bubbling with enthusiasm, fervour, and passionate love for mathematics. When he was giving a lecture, he looked restless, pacing up and down, trying to convey to his audience the beauty and order he saw in mathematics. His loud and clear voice, speed, and complete command over the subject matter of his talk kept his audience spellbound. To him mathematics was more than a subject; it was his very life. He loved, breathed, and tasted mathematics and above all, he tried to make others infected with it, too. To his students he was more than a teacher, he was a guru in the true sense. To his colleagues he was an idealist, with no room for compromise. There were only two ways—the right and the wrong, and the right won everytime. To his admirers he was almost superhuman and he could do no wrong.

## BIRTH AND EARLY EDUCATION

Prabhu Lal Bhatnagar was born on August 8, 1912 in Kota in Rajasthan, the second of five sons. His parents belonged to a well connected family, which had been advisers to the rulers of the princely state of Kota. His forefathers were given the name 'Rai Dwarkdas' and had donated all their wealth and property to the Lord Mathuradhish temple at Kota. His first lessons in arithmetic were from his grandfather, who enjoyed giving him problems to work mentally. Young Bhatnagar showed promise from the very beginning and nothing came between him and his studies. He went to school first at Kota, then the Government school at Rampura and Herberter College, Kota. He had secured first rank in the Intermediate examination and was encouraged by the Principal and Director of Education, Lala Daya Kishan Gupta, to go for further studies to Jaipur. It was during this time that his father passed away. Bhatnagar was just over twenty years old and a regular recipient of a scholarship from matriculation onwards. With this he supported himself and his family. He was a man of simple ways throughout his life. His true genius became further evident in 1934, when he completed his BSc degree at Maharajah's College, Jaipur. It was then affiliated along with other colleges to Agra University. Bhatnagar had obtained the highest marks that year in Mathematics, and Chemistry in the BSc examination of

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Agra University and for this he was awarded the Krishna Kumari Devi gold medal and Umang Lakshmi Kanti Lal Pandya gold medal, respectively. For the first rank in BSc, he was awarded the Maharajah Fateh Singh gold medal.

## Higher Education and Research Activities

Bhatnagar went on to do his MSc at the same college. Professor KL Varma was one of the teachers for whom Bhatnagar had profound regard. In 1936 for his outstanding result in the MA, MSc examinations of Agra University, Bhatnagar was awarded the Lord Northbrook gold medal by Maharajah's College, Jaipur. The young Bhatnagar must have come to the crossroads at this stage. All brilliant young men of that time aspired to appear at the ICS examination. It was a lucrative, powerful profession for a chosen few. However, Bhatnagar was no ordinary young man. In spite of pressures and coaxing from friends and relatives, he turned down the ICS in favour of the advice of his teacher, Professor KL Varma. Professor Varma had advised him to take up research as a career at Allahabad University.

### *At University of Allahabad*

Allahabad University was at that time at the pinnacle of its glory. Professor AC Banerji, a Wrangler from Cambridge, was the Head of the Department of Mathematics at that time. Bhatnagar first worked with Professor BN Prasad on the summability of Fourier and Allied Series. His research work at Allahabad from 1937-39, included the solution of second order linear ordinary differential equations by the Laplace Transform technique. Two of these have been incorporated in the famous book of Kamke *Differentialgleichungen, vol. I.* These are

(i) $y'' = (a^2x^{2n} - 1)y.$

Solution for $anx^{n-1} > 1$ is

$$y = C_1 \int_{-1}^{1} + C_2 \int_{1}^{\infty} e^{-\lambda t} (t-1)^{\mu-\nu} (t+1)^{\nu+\mu}\, dt,$$

$$\lambda = \frac{a}{n+1} x^{n+1}, \quad \mu = -\frac{n+2}{2(n+1)}, \quad \nu = \frac{x^{1-n}}{2a(n+1)}$$

and

(ii) $y'' = (4a^2 b^2 x^2 e^{2bx^2} - 1)\, y$

with solution

$$y = \int e^{-\xi t} (t-1)^{\eta-\zeta} (t+1)^{\eta+\zeta}\, dt,$$

where $\xi = ae^{bx^2}, \quad \eta = \dfrac{1-2bx^2}{4bx^2}, \quad \zeta = \dfrac{e^{-bx^2}}{8ab^2x^2}$

and the limits of integration are so chosen, that at these points, the integrand multiplied by $t^2 - 1$ must vanish.

Both these results were published in collaboration with Professor AC Banerji in the *Proc Nat Acad Sci* 8 1938.

Bhatnagar's research interests slowly shifted to the area of astrophysics. This was an inspiration on coming into contact with Professor MN Saha, FNA, FRS, who was already famous for his work in physics and astrophysics. Bhatnagar began to work on the spiral nebula and the tidal theory of planetary formation. For the best research in the Faculty of Science, Allahabad University, during 1937-39 he won the EG Hill memorial prize. He obtained his DPhil degree in Mathematics in 1939 for his thesis entitled *On the origin of the solar system* under the supervision of Professor AC Banerji.

*At University of Delhi*

After obtaining the DPhil degree of Allahabad University, he was invited by Shri SN Mukherjee, the then Principal of St Stephen's College, Delhi to join the college. Here he spent the better part of the next 16 years of his life, first as a Lecturer and later as head of the Department of Mathematics and concurrently a Reader in Mathematics at Delhi University. These years could perhaps be termed the blossoming years when Bhatnagar was in full bloom. He indulged in his interest in astrophysics, working both independently and in collaboration with Professor DS Kothari. The result was a spate of publications from 1939 to 1946, the highlight of which was the theory of anharmonic pulsations of Cepheids and white dwarf stars. This work brought Bhatnagar international recognition and caught the attention of the scientific community, especially the astronomers.

While the pulsation phenomenon had been observed only in 'supergiant stars', Bhatnagar saw no theoretical reason why it should not occur in denser stars and even in white dwarfs. He felt that a nova outburst associated with the sudden collapse of a star could leave the white dwarf pulsating. The pulsation once started could last for a period comparable to $10^3$ years. The period of pulsation (assuming a 'homogeneous model') for the fundamental mode is given by

$$P = \left\{\frac{9h^3}{16\,\pi\, m^{3/2}\, H^{5/2}\, G^2}\right\} \frac{1}{\mu^{5/2}\, M} \sim 10\, \frac{\odot}{M} \text{ sec.}$$

where $M$ is the mass of the white dwarf, $h$ the Planck's constant, $m$ the electron mass, $H$ the proton mass, $G$ the gravitational constant, $\mu$ the mean molecular weight, and $\odot$ the solar mass. However, Bhatnagar observed that the pulsation period for a white dwarf was too small to be directly observable and so the existence of pulsation in white dwarfs had to be looked for through its secondary effects. In later years, high speed techniques have discovered such pulsations and substantially verified his calculations.

In an attempt to explain the observed skewness in the velocity time curve of the Cepheid Variables, Bhatnagar and Kothari derived exact expressions for the period $P$ of oscillation and the times $t_1$ and $t_2$, which are parts of the period, where the radius of the star is greater and less than the equilibrium radius $R$, respectively. They observed that if $\gamma$, the ratio of specific heats, were taken to be 5/3, then the observed skewness would demand a semiamplitude of oscillation which was almost equal to the radius of the star. This was inconsistent with observation, which gave a value for the semiamplitude as 0.1R. A rough calculation showed that the observed skewness would arise for a semiamplitude of 0.1R only if $\gamma$ were comparable to 10. The complex computer calculations of Robert Christy later showed that the skewness was essentially a surface phenomenon and the equations used by Bhatnagar were too slowly convergent to handle surface layers.

At St Stephen's, a busy schedule of over 20 hours of teaching per week did not dampen Bhatnagar's enthusiasm for research. In 1947 he was awarded the DSc degree from Allahabad University for his work on Astrophysics. His interest in stellar structures and interiors led him to the study of rarefied gases and ionised media. This was a harbinger of the monumental work he was to do a few years later.

*At University of Harvard*

In 1951, Bhatnagar went to Harvard University, Cambridge, Massachusetts as a Fulbright scholar for two years. This handsome tall scholar from India was often mistaken in the University corridors for a student. Once he took his place at the lecture rostrum, the students realized that he was indeed a senior faculty! He lectured on the mathematical theory of non-uniform gases. Together with DH Menzel and HK Sen, he wrote a book *Stellar Interiors*, which was published in the International Astrophysics Series, Chapman and Hall. The Boltzmann equation had captured Bhatnagar's attention at that time. The complicated integrals, which gave the collision effects were far too difficult to handle. His passion for simplification led to the emergence of the B-G-K (Bhatnagar, Gross, Krook) model. This gives a simple, yet a very realistic, Boltzmann-like equation, which has since been used as an alternative to the Boltzmann equation in actually solving problems in rarefied gas dynamics, plasma physics, and the kinetic theory itself. The classic paper of Bhatnagar, Gross, and Krook in the 94th volume of *Physical Review* of 1954 is the most widely referred paper in plasma physics and is still very extensively used.

There are two ways to describe the behaviour of a gas : (1) the continuum theory, where the macro (or bulk) behaviour is described and (2) the kinetic theory, where the behaviour is determined by the micro structure of the the constituent molecules. Here, a gas in a confined space is regarded as a swarm of randomly moving particles, which perform collisions, and whose motion is heat itself. In continuum theory, the Navier-Stokes equations govern the motion of a gas in bulk. In kinetic theory, it is possible to derive laws governing the motion of a gas, which in a suitable limit lead to the Navier-Stokes equations. When the density of a gas becomes sufficiently low and the mean free path of molecules is no longer negligibly small compared to a characteristic dimension of the flow geometry, results of continuum fluid dynamics require correction. This becomes more and more important as the degree of rarefaction increases. Continuum dynamics must then be replaced by the kinetic theory of gases, and the Navier—Stokes equations by the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} = Q(f,f), \tag{1}$$

which gives the evolution of an "expected number density" $f(\mathbf{x}, \mathbf{v}, t)$ in the phase space described by $(\mathbf{x}, \mathbf{v})$, i.e. $f(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}d\mathbf{x}$ is the "expected number of molecules" which at time $t$ lie in the "volume" element between $(\mathbf{x}, \mathbf{v})$ and $(\mathbf{x} + d\mathbf{x}, \mathbf{v} + d\mathbf{v})$ in the phase space, $Q(f, f)$, called a collision term, is given by

$$Q(f,f) = \int \{f(\mathbf{v}')f(\mathbf{v}_1') - f(\mathbf{v})f(\mathbf{v}_1)\} \mid \mathbf{v}_1 - \mathbf{v} \mid \mathrm{r}d\mathrm{r}d \in d\mathbf{v}_1 \tag{2}$$

and represents the time rate of increase of the "expected number" of molecules (per unit "volume" in the phase space) due to binary collision of the molecules. In (2) the dependence of $f$ on the velocities only has been shown; its dependence on $\mathbf{x}$ and $t$ has been suppressed. $\mathbf{v}'$ and $\mathbf{v}_1'$ are the velocities of two colliding molecules, which after collision have velocities $\mathbf{v}$ and $\mathbf{v}_1$. $(r, \in)$ represents the plane polar coordinates in the plane of collision, with the particle which comes out with volicity $\mathbf{v}$, as the centre of the coordinate system. For the validity of the Boltzmann equation, it is not necessary to assume the gas flow to be rarefied. It is valid for much denser gases and has been used to compute the transport coefficients, namely, the coefficients of viscosity, diffusion and thermal conduction, in order to complete the conservation equations of continuum mechanics.

The collision term $Q$ has quadratic nonlinearity and involves a very complicated integral. Hence the Boltzmann equation is an extremely difficult nonliner integro-differential equation, which had evaded all attempts at a solution. Boltzmann equation led to some really deep results. One of them is : A quantity $H$ associated with total gas is a non-increasing fuction of time (the celebrated H-theorem of Boltzmann) and is steady only when the distribution function $f$ is Maxwellian. Another result is to be noted regarding the collision processes of two molecules, which is completed over a very short distance compared to their mean free path. After the collision, the total mass, momentum and thermal energy (simply the kinetic energy for simplest molecules, i.e. molecules for which the state can be described by three space coordinates and three velocity components) of the molecules remain the same as those before the collision.

Thus, for any collision process, there are five independent functions $\psi_\alpha$ (**v**), $\alpha = 0, 1, 2, 3, 4$, such that $\int \psi_\alpha(\mathbf{v})\, Q(f,f)\, d\mathbf{v} = 0$, where the integration is over the whole three dimensional velocity space **v**. Hence $\psi_0 = 1$, $(\psi_1, \psi_2, \psi_3) = \mathbf{v}$ and $\psi_4 = v^2$. The functions $\psi_\alpha$ are called elementary collision invariants and it can be proved that a general collision invariant for the simplest molecules is a linear combination of the above mentioned five elementary collision invariants.

One of the major shortcomings in dealing with the Boltzmann equation is the complicated nature of the collision integral. Bhatnagar (along with his collaborators, Gross and Krook) realized that much of the detail of the two-body interaction, which is contained in the collision term, is not likely to influence significantly the values of many experimentally measured quantities and coarser description may be obtained by replacing the true collision term $Q(f,f)$ by a simpler one $J(f)$, which retains only the qualitative and average properties of $Q(f,f)$. This leads to the creation of the BGK model in which $J(f)$ is assumed to possess the following two main features of the true collision terms $Q(f,f)$ : (1) The elementary collision invariants of $Q(f,f)$ retain their properties with respect to $J(f)$ also, i.e. $\int \psi_\alpha J(f)\, d\mathbf{v} = 0$, $\alpha = 0, 1, 2, 3, 4$. (ii) The collision term expresses the tendency to approach a Maxwellian distribution (H-theorem).

Bhatnagar, Gross and Krook took the second feature into account by assuming that each collision changes the distribution function $f(\mathbf{v})$ by an amount proportional to the departure of $f(\mathbf{v})$ from the Maxwellian $\Phi(\mathbf{v})$, i.e. $J(\mathbf{v}) = \nu\,[\Phi(\mathbf{v}) - f(\mathbf{v})]$, where $\nu$ is independent of **v** and the other parameters in $\Phi(\mathbf{v})$ are determined from the fact that at and point **x** any time $t$, $\Phi(\mathbf{v})$ must have the same density, velocity and temperature of the gas as that given by $f(\mathbf{v})$. Thus unlike the quadratic nonlinearity of $Q(f,f)$, the nonlinearity in BGK collision term is much more complex.

The main advantage in using the BGK operator is that for any problem one can deduce integral equations for the macroscopic variables : density, velocity, and temperature. These equations, though strongly nonlinear, simplify some iterative procedures and make the treatment of interesting problems feasible on high speed computers. A problem, which is easily solved with the nonlinear BGK model, is that of the relaxation to equilibrium in the spatially homogeneous case. An arbitrary distribution function $g(\mathbf{v})$ depending only on the velocity **v** is given, and we want to find its time evolution according to kinetic theory; this problem cannot be solved analytically with the full Boltzmann equation, but it is trivial with the BGK model. Another simple property of the BGK model is that it admits an easily proved H-theorem.

BGK model contains the most basic features of the Boltzmann collision integral, however it is not derived from the Boltzmann equation. Therefore, the discovery of the BGK model has led to systematic investigations for deducing models of increasing accuracy. Undoubtedly, the BGK model is one of the most important scientific contributions by an Indian mathematician.

*Return to Delhi University*

On his return to India, Bhatnagar exemplified two of his convictions, which he held throughout his life. The first was to reach out to as many people as he could, and help them pursue research. He believed that since he had the capacity and the gift to do independent research, he should inspire others and introduce them to the joys of scientific research. In his own words, '*Reward of research is the joy of creation.* Besides, every scientist had social obligations to fulfil. The second of his convictions was that the research of tomorrow has to be done by a group. The more people there are working cohesively on various aspects of the same theme, the more fruitful the findings would be. In 1953, Bhatnagar began to gather a band of colleagues and other teachers from Delhi University and encouraged them to take up research. As local secretary of the annual conference of the Indian Mathematical Society in December 1953 at Delhi, he used every opportunity to draw others into the scientific fold. Just as the scientific activity was picking up, there was a change in Bhatnagar's life.

*At IISc, Bangalore*

In 1950 he was elected fellow of the National Institute of Sciences (now INSA, Indian National Science Academy). In 1955, he was elected fellow of the Indian Academy of Sciences. As his stature rose, he was much sought after as a Professor. In January 1956, the Indian Institute of Science, Bangalore invited him to join as the first Professor of the newly created Department of Applied Mathematics. The venue of Bhatnagar's life and research moved from Delhi to Bangalore. It was initially intended that Bhatnagar give lectures on various topics in mathematics in the already established departments of science and engineering at the Institute and that the scientific community could benefit from his presence at the Institute. It was not intended that the Department grow in size or scope, but that it remain very much a one-man affair.

This, however, was against all that Bhatnagar believed in or cherished. He was bubbling with enthusiasm for scientific research and he had to transmit this to others. Slowly he began to gather young research students from all parts of India. In order that the department be truly a department of applied mathematics, Bhatnagar realized he would have to widen his scientific research. Besides kinetic theory of gases and plasma physics, he initiated research in fluid dynamics, including boundary layer theory, magnetohydrodynamics, and the theory of non-Newtonian fluids. The choice of these subjects was inspired by the fact that the engineering departments at the Institute could verify experimentally the theoretical results obtained by his group. He desired that as far as possible, research activity at the Institute should be a cooperative effort. He was also acutely aware of the fact that it would be nonlinear effects and discontinuous solutions that would be the work of the future and he initiated work in shock propagation and the theory of nonlinear waves. He was also convinced that computers would play a dominating role in research in the years to come and encouraged the use of computer oriented numerical techniques and the study of mathematical logic along with other basic mathematical topics, like group theory and Boolean algebra.

Special mention should be made of Bhatnagar's work in the area of non-Newtonian fluid flows, as it was a relatively new area at that time and many questions were yet to be answered. The famous experiments of Weissenberg and Merrington had shown that fluid behaviour could not always be explained by the Newtonian stress-rate-of-strain relation :

$$\tau_{ij} = - p\, \delta_{ij} + \mu e_{ij},$$

where $\tau_{ij}$ is the stress tensor, $e_{ij}$ is the rate-of-strain tensor, $\delta_{ij}$ is the Kronecker delta, p is the pressure and $\mu$ is the Newtonian coefficient of viscosity. When a fluid is subjected to shearing flow between two coaxial cylinders, the inner of which is at rest and the outer rotating, then, in general, the level of the free surface close to the inner cylinder drops. However, in certain highly viscous fluids, it was found that the fluid close to the inner cylinder was forced inwards against the centrifugal force and upwards against the force of gravity, so that the fluid rose at the inner cylinder. The experiment showed that in addition to the shear stress components, which were predicted by the Newtonian theory, there were additional normal stress components not predicted by the theory. This led to a spate of fluid models generally referred to as "non-Newtonian" fluids. Among these were models : (i) those which took account of the elasticity of the fluids—"viscoelastic fluids", (ii) those which were governed by higher order stress-rate-of-strain relations—"power law fluids"—, (iii) those with cross viscosity, (iv) those with couple stress and microrotations. In order to classify and distinguish between various non-Newtonian fluid models and calculate the innumerable constants associated with these models, Bhatnagar undertook a study of secondary flows. He realized the importance of studying shearing flows in these fluids and together with his doctoral students published a large number of papers on flows between rotating boundaries, like flat plates, cylinders, spheres, cone and plate, etc. Truesdell and Noll (1965) in their book *The nonlinear field of theories of mechanics* (Handbuch der Physik, Band III) have mentioned : *Bhatnagar and his collaborators have calculated and classified the secondary flow corresponding to various geometrical conditions.*

Besides suggesting how to classify various non-Newtonian fluids, Bhatnagar also realized that these different models, though apparently based on widely differing phenomenological considerations, behaved very similarly in shearing flows. In fact, the Oldroyd model, the Walter model and the Rivlin-Ericksen model (with a certain restriction on its parameters) could be described in shearing flow by the same expressions with a suitably defined common non-Newtonian parameter. Bhatnagar also showed that the normal stresses, which were present in rotating shearing flows of non-Newtonian fluids opposed the centrifugal forces, in general. This leads to a phenomenon peculiar to non-Newtonian fluids, namely, even when the boundary rotates in the same sense, the secondary flows can "break" and there can be reversal of flows, provided the normal stresses are strong enough to overcome the effects of the centrifugal forces.

In the period 1960-1965, Bhatnagar spent many months abroad at the Harvard College Observatory at Cambridge, Mass. During this period, Bhatnagar developed a severe problem in the spinal region of the lower back. He was unable to walk long distances or keep standing for a long time. That did not dampen his spirit. In spite of the fact that he usually paced up and down impatiently during a lecture, he got used to delivering talks while seated in a chair. The walk to the department and back to his house was a painful process and he would have to rest in between. It was a familiar sight to see him seated on the steps en route with a knot of students around him, discussing some research problem quite unmindful of the pain in his back. Surgery in the United States corrected his spinal problem and soon *he was seen to run and dash* around on campus.

*At Rajasthan University at Jaipur*

By April 1969, the Department of Applied Mathematics at the Indian Institute of Science, Bangalore had grown in stature. About 25 of his students had either completed or were in the process of completing their work for a PhD degree. The department had grown in size to over ten faculty members working in various areas of applied mathematics. Bhatnagar was not one to rest on his laurels. With an overwhelming desire to serve a larger academic community, he decided to leave it all all behind and move to Jaipur, where he was offered the Vice-Chancellorship of the University of Rajasthan. It was a time of student unrest and tumult. Bhatnagar with his strong belief in the innate honesty and goodness of man, especially of the younger generation, was never ruffled. He talked to the students, cajoled, coaxed and convinced them on major disturbing issues. He prided himself on the fact that anyone could walk into his office and talk to him. However, this was not his life's work and it pained him that he found very little time for research; so he decided to relinquish office at the end of two years.

*At Himachal Pradesh University*

In May 1971, at the invitation of the newly formed Himachal Pradesh University at Shimla, he joined as Senior Professor and Head of the Department of Mathematics. Soon after, he left to spend a year as a Visiting Professor at the University of Waterloo in Canada. It was a special occasion, because for the first time he was going abroad with his wife, Anand Kumari, his helpmate for over forty years. To him family always meant the "extended" family—his students, colleagues and coworkers included. His wife and children hardly saw him during the week, because there were so many of his 'extended' family clamouring for his time and attention. It was always 'open house' at the Bhatnagars, with gracious hospitality.

At Shimla, Bhatnagar again set out from scratch to build a department of mathematics. He was a UGC National Lecturer in 1972. He also gave the NR Sen memorial lecture sponsored by the Calcutta Mathematical Society. It was while he was at Meerut on a lecture assignment in January 1973 that he heard about the passing away of his wife at Kota. After this tragic incident, Bhatnager was very unhappy and alone at Shimla. His appointment as member of the Union Public Service Commission came as a welcome change and he moved to Delhi in October 1973.

It was a large house close to India Gate, that Bhatnagar lived in. However, he was very lonely. Of his four sons, Rakesh and Brijendra were abroad and the others, Vinay and Kamal, working outside Delhi. His only daughter, Kalpana, was busy with her own growing family at Bhopal. As usual, he devoted his entire energy to his work. At heart, he was a mathematician and a teacher and UPSC could not hold him long. When the Mehta Research Institute (MRI) of Mathematics and Mathematical Physics at Allahabad was to take shape, Bhatnagar accepted the offer to be its first Director in July 1975.

*At Mehta Research Institute*

For a brief spell, Bhatnagar was his old self. With dreams of the future, he worked ceaselessly to build a true centre of research in Mathematics. His magnetic personality attracted to MRI people of the highest calibre. For many years he had been devoting his time to developing curricula for mathematics education.

In December 1975 he convened a Regional Conference on the Development of integrated Curriculum in Mathematics for developing countries in Asia. The conference was at Bharwari, near Allahabad. MRI was still in its infancy. Bhatnagar conducted the proceedings with finesse—looking after the foreign and Indian guests, arranging the lectures, initiating discussions—all in this tiny little place. It was only he, who could have done it.

In May 1976, the indefatigable Bhatnagar conducted a one month course on 'Hyperbolic Systems of partial differential equations and nonlinear waves'. With only one faculty member to help him, he kept up a furious pace of lectures starting from a simple linear wave, going through the Burger's and KdV equations in detail, the method of inverse scattering, group velocity in nonlinear systems and its Lagrangian formulations. His lecture notes at this course formed the subject of his book

*An introductory Course on Nonlinear Waves* which appeared as an Oxford Mathematical monograph.

## Association with learned Societies/International honours

In 1950, Bhatnagar was elected Fellow of the National Institute of Sciences of India (now INSA, Indian National Science Academy). In 1955 he was elected Fellow of the Indian Acadeny of Sciences. As the tempo of his research activities increased, Bhatnagar began to get more and more international recognition. He was a member of Commissions 27 (Variable stars) and 43 (MHD) of International Astronomical Union, and a corres ponding member of the International commission on Plasma Physics, appointed by the International Union of Pure and Applied Physics. In 1967 he was invited by the Royal Society as a Distinguished Visiting Scientist and he delivered two lectures on his research work on *Slip Flows* and *Secondary Flows* at the Imperial College, London. In 1968 he was invited to the International Congress on Rheology at Kyoto in Japan.

At home, too, the 1960's was a very busy period for Bhatnagar. He was among other things, President of the Mathematics Section of the Indian Science Congress (1962), the Indian Mathematical Society (1964, 65, 68), the Physical Sciences Section of the National Academy of Sciences (1969), the Congress of the Indian Society for Theoretical and Applied Mechanics (1971), and the Association of Mathematics Teachers of India (1968-76). In his immediate neighbourhood, he formed the Bangalore Mathematical Association and under its auspices, apart from lectures etc., introduced Mathematict Olympiads on the lines of those held in East European Countries. This was the first ever Mathematics Olympiad held in India and it is a tribute to the foresight of Bhatnagar that almost twenty years ahead he saw the need for these olympiads in India. Today the National Board for Higher Mathematics (NBHM) conducts the Indian National Mathematics Olympiad (INMO) all over India to detect and nurture talent in mathematics among students at the level of standard XI. For his service to the nation, Bhatnagar was fittingly awarded the Padma Bhushan on January 26, 1968.

He was an UNESCO expert in the United Arab Republic for a three month period in 1969, a member of the Executive Council of the International Conference on Mathematics Education at Lyon in 1969, a leader of the Indian delegation to the Indo-US Binational Conference on Mathematics Education and Research at Bangalore in June 1973, and a member of the International Commission on Mathematical Instruction at Vancouver in August 1974.

In spite of recurring spells of pain and giddiness, Bhatnager went abroad in 1976. He had been leader of the Indian delegation to the XIV International Congress of Theoretical and Applied Mechanics at Moscow in 1972 and was now attending the

XV Congress at the Hague, Netherlands. He also attended the International Conference on Mathematics Education at Karlsruhe in West Germany.

## QUALITIES AS A MAN

In all that is said of Bhatnagar 'the scientist', it is incorrect to overlook Bhatnagar 'the man'. He was a man of simple ways and high principles. He believed in the existence of God, but at the same time, he felt that this belief was not wholly rational. The rituals of 'bribing' deities for purely selfish reasons were obnoxious to him. Even the thought of bargaining with his employers for his own benefit was repulsive. He said *I would not like to bargain with an Institution that I am going to serve.* All his life he indulged in writing poetry. He never published his poems, but those who have heard him recite his poetry were impressed by his creativity and found it an unforgettable experience. Humility was another of his characteristics. If he found that he were in the wrong, he never hesitated to apologize to his students or anyone else when he felt he had wronged. Perhaps the following words of Bhatnagar summarize his qualities as a human being : *What I cherish most in my life is peace and humility. Peace within ensures peace outside and wise decisions in all matters. Humility enables one to understand the other man's point of view and creates an atmosphere of affection all around you.*

### *End of a glorious life*

A month after his return from abroad to India on October 5, 1976 Bhatnagar passed away very quietly. He was to go to Delhi that afternoon. He had complained of chest pain and had gone for a medical check-up at 9.30 am. The doctors found nothing wrong with him and even told him that he could go to Delhi that afternoon. A few minutes after leaving the hospital, alone in the back seat of his car, he had a massive heart attack and passed away. His body was consigned to the flames on the banks of the Ganga.

A man of his calibre needs no epitaph. His life is an example in itself and speaks far more than what anybody can write about him. His was a life of simple honesty, high ideals, and high thinking. May his tribe increase.

VG TIKEKAR, PHOOLAN PRASAD, RENU RAVINDRAN

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