## SUDDHODAN GHOSH

## (1896–1976)

## Elected F.N.I. 1951

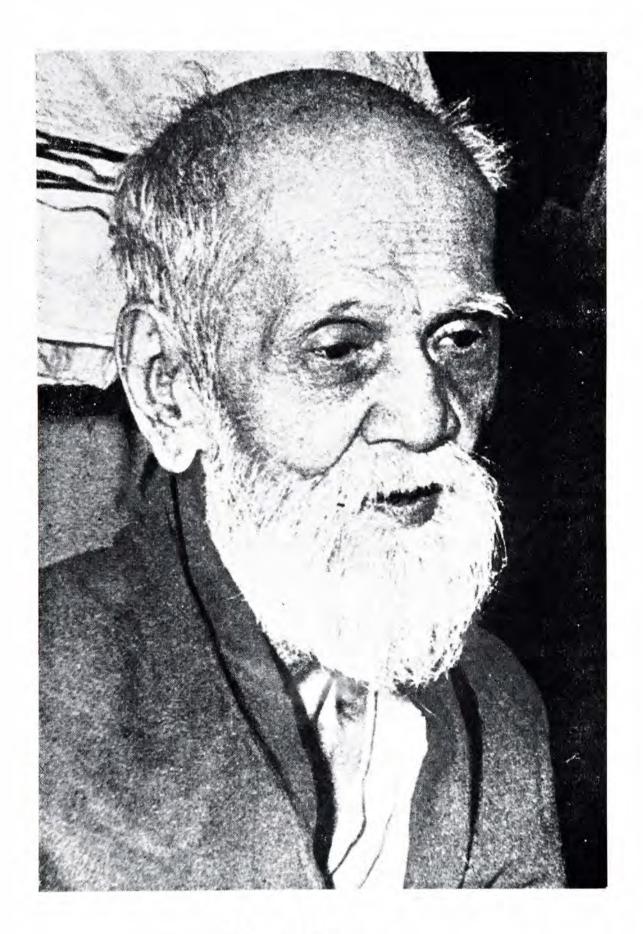
### BIRTH, PARENTAGE AND CHILDHOOD

SUDDHODAN GHOSH was born in Calcutta on July 26, 1896, with a silver spoon in his mouth. He belonged to an aristocratic Kayastha family of Bengal. In days of yore, his ancestors lived in Gobindapur, in the vicinity of Fort William. With the advent of the British, the family shifted to Behala and thence to Jorasanko, near the palace of the Tagore family. They built a castle there. When baby Suddhodan first opened his eyes, he saw a world of sunshine and splendour. His father, Sarat Chandra Ghosh was an affluent landlord, while his mother Patitpabani Debi, was a generous and devout lady. They had twelve children five sons and seven daughters. Suddhodan was the fourth son and eleventh child.

Ghosh's grandfather, Harachandra Ghosh, was a prominent personality. A contemporary of the eminent littérateur, Bankim Chandra Chatterjee, he was in the first batch of students to graduate in Arts from the Calcutta University. He was also the first Bengali Chief Judge of the Calcutta Small Cases Court, where his marble bust still adorns its courtyard. Harachandra's eldest son, Pratap Chandra Ghosh, assumed reins of the family. A strict disciplinarian, he was also an orthodox puritan. He exerted a firm but benign influence over the family. Young Suddhodan was deeply attached to his uncle for his truth, application and rectitude of conduct.

When asked about his early life, a montage of childhood memories flashed through his mind. He narrated : "I was born a weakling. My delicate constitution did not permit me to participate in any healthy pastime. I could hardly run a few yards for want of breath. The house physician prophesied that I would not outlive a dozen years. Thus, I became my mother's pet and pampered child. Nobody scolded me. Thanks to my mother's prayers and penance and through God's grace, I outwitted the doctors and developed into a normal child. Books became my delight, my one absorbing passion. With a passion for books, I developed a love of solitude. Alone and aloof from the excitements of the world, I delved into books and scriptures, watching things profane and things divine from a distance."





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## FORMATIVE INFLUENCE DURING EARLY LIFE

The Ghosh family was large. Their residential palace accommodated more than a hundred inmates. It included the joint families of Pratap Chandra Ghosh and his two brothers, their progenies, retinues and guests. The days were crowded with ceremonies, religious festivities, feeding destitutes and giving alms to the needy. Thus, as a boy, young Suddhodan secured first-hand experience of the joy associated with perpetual giving and expecting nothing in return.

He also regularly attended the learned discourses of Sanskrit Pandits and renowned scholars, who congregated at his home. Buddhist pilgrims from Tibet and the far east were spontaneous visitors. The essence of Buddhism and Jainism found a responsive echo in his heart. Mahamahopadhyay Chandrakanta Tarkalankar was a neighbour and family friend. He related mythological stories from the Ramayana and Mahabharata to the children. The legend of Bhishma and his sacrifices set young Suddhodan's heart aflame. He took a vow to follow his footsteps, and verily, he did succeed. The blow of circumstances shaped his character from the rock while his soul acquired lustre and purity in the refiner's fire.

In his youth, Suddhodan Ghosh witnessed the emergence of the golden era of Bengal. The leaders were Vivekananda, Aurobindo, Rabindranath Tagore, Surendranath Banerjee, Chittaranjan Das and others. They dreamt about the glorious future of the Motherland. Suddhodan Ghosh was drawn into these visionaries' vortex. In his early youth, he adored Tagore. He attended every single meeting addressed by him. He often recited the following lines from "Gitanjali".

"Where the mind is without fear

And the head is held high;

Where knowledge is free;

Where the world has not been broken up

Into fragments by narrow domestic walls;

Where words come out from the depth of Truth

Where tireless striving stretches its arms towards perfection;

Where the clear stream of reason has not lost its way into the dreary sand of dead habit;

Where the mind is led forward by Thee into ever-widening thought and action-Into that heaven of freedom,

My Father, let my country awake."

During later years, Mahatma Gandhi cast a spell over him. He joined the Non-Cooperation movement. He courted imprisonment in 1921, and spent three months in jail for participating in the struggle for national independence. He lived a simple, austere and dedicated life. He wore only Khadi clothes and garments, made from self-spun cotton fibres. Charkha spinning was his daily ritual. He established close association with self-supporting organisations like Abhoy Ashram, Khadi Pratisthan etc. However, the internal strife within the Congress Party distressed him. With Gandhiji's permission, he left the Congress Party. He



ceased to be a politician, but remained a true nationalist to his last breath. Ironically enough, Gandhiji, himself, ceased to be a member of the Congress Party years later.

Acharya P. C. Ray was his distant uncle. He was a bachelor and lived a life of self-denial. The same simplicity and naturalness also characterised the dress and manners of Suddhodan Ghosh. He also reduced his diet to the minimum, necessary for the preservation of life. He could not but strike a casual visitor with awe and reverence due to his christ-like appearance and saintly character.

# SCHOOL AND UNIVERSITY EDUCATION; PRELIMINARY RESEARCH

Suddhodan Ghosh took to reading, writing and Arithmetic during his infancy, just as a duckling takes to water. He became a child prodigy. Mathematics was his forte.

He took his admission in Vidyasagar's Metropolitan Institution as a student of the eighth (present day third) class. Soon he outshone all his rivals. In quest of keener competition, he sought admission into the Hindu School with its hoary tradition. His grandfather Harachandra Ghosh happened to be one of the founder-donors of the Hindu school. But young Suddhodan refused to seek admission as a donor's candidate. He was admitted to the fourth (present day seventh) class in 1910 strictly on merit. He topped the list of successful candidates. He became a favourite of the renowned mathematics teacher Upendra Lal Bakshi. Often he was called upon by his teacher to go to the blackboard in the next higher class and solve difficult problems which his seniors could not tackle. Sri Dilip Kumar Roy, the renowned singer, was one year senior to Ghosh.

In 1914, Suddhodan Ghosh passed the Matriculation examination in the first division, getting a place near the tenth position in order of merit.

Soon an atmosphere of gloom descended over the Ghosh family. The head of the family, Pratap Chandra Ghosh, expired. His youngest brother embraced Christianity. The joint family disintegrated. There was a touch of sadness in everything. During this period, Suddhodan Ghosh joined the Intermediate-in-Science class of the Scottish Church College. In the annual examination for promotion into the second year class, he topped the list of successful candidates. He became eligible for a scholarship, but there was a precondition attached to the award. The scholar had to sign a declaration that he came from a poor family. Suddhodan hesitated to sign the form, because he could not acknowledge that his father was indigent. The rector was agreeably surprised by the boy's forthrightness. He gave him a day to reconsider his decision, but Suddhodan remained adamant. Next day, he reiterated his earlier decision. Consequently, the second boy got the scholarship. In the joint Intermediate final examination (I. A. & I. Sc. together) of the Calcutta University in 1916, he secured the eighth place in the merit list. Incidentally, he got the highest marks in Mathematics.

He joined the third-year B.Sc. class (with Honours in Mathematics) of the Presidency College, Calcutta. During this period he had to face financial difficulties. One of his aunts, who had been married into a rich family, came to his rescue. She gave him enough money to purchase his books and a wooden



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almirah to store them. This almirah remained in his possession till the last. Now it has been preserved in the Bangiya Bijnan Parishad with the relics of Dr Ghosh. It is needless to add that his aunt's loan was promptly repaid after he got the scholarship.

With his joining the Presidency College, began the golden era of his academic career. He came under the tutelage of eminent teachers like Acharya J. C. Bose, Acharya P. C. Ray and Dr D. N. Mallik and blossomed into a top-ranking scholar. He stood first with first class in the B.Sc. (with Mathematics Honours) examination of the Calcutta University in 1918. He was awarded the University Gold Medal for his brilliant performance. He took the medal home and placed it at the feet of his mother, who was visibly moved. She reciprocated his gesture by presenting him with a gold mohur (Akbari era), which she had earned from Pratap Chandra Ghosh as a reward for her exquisite floor decorations. Dr Ghosh cherished this mohur as his greatest treasure. After his demise, it was gifted to the Calcutta University museum, in deference to his last wish.

He joined the University College of Science and Technology at 92, Upper Circular Road, Calcutta in 1918 as a post-graduate student of Mixed Mathematics. He studied Mathematical Theory of Elasticity with Professor S. N. Bose, who was then lecturer in the department. He passed the M.Sc. examination in 1920 standing first in first class, securing about 90 per cent marks, earning the University Gold Medal. Unfortunately, his mother did not live to witness the presentation of the second award, she had expired a year earlier.

Next, he worked as a research scholar under the guidance of Professor Nikhil Ranjan Sen, then Sir Rash Behari Ghosh Professor of Applied Mathematics.

### PROFESSIONAL CAREER

After obtaining the topmost rank in the M.Sc. examination in Mixed Mathematics in 1920, Suddhodan Ghosh was offered a prized post at the Imperial Bank of India. Unhesitatingly, he declined the offer, since he did not wish to abandon mathematics. Sometime later, Sir P. C. Ray offered to obtain for him a scholarship for advanced study abroad. This too, he declined. He did not like to leave his Motherland. He felt that while poverty and struggle for existence sometimes dampened intellectual ardour, wealth and affluence invariably did so. The most suitable condition for a scholarly life according to him, was one above want-not exactly affluence, but immunity from petty worries of life, security against the uncertainty of the morrow. And so, he became Sir Rash Behari Ghosh Research Scholar with a pittance of Rs. 75/- per month. In 1925, he competed for and won the Premchand Roychand Studentship Prize of the Calcutta University-the most coveted award at the disposal of the University. He also won the prestigious Sir Asutosh Mookerjee Gold Medal of the Calcutta University for his outstanding researches in Mathematics.

After obtaining the D.Sc. degree of the Calcutta University in 1928, he was persuaded by Professor S. N. Bose to join the Dacca University as Lecturer in Mathematics. But, after a brief stay at Dacca, he returned to his *alma mater* and



joined the Department of Mixed Mathematics. Soon he was acclaimed as a great teacher and top academician by his pupils and colleagues.

In 1953, the Calcutta University selected Dr Suddhodan Ghosh as the first Reader in the Department of Applied Mathematics. He, however, declined to accept the appointment, since he felt he was superseding a senior colleague. The university authorities were non-plussed; however, they averted the complication by transferring the Readership to a sister department.

A somehwat similar situation arose in 1954, when Sir J. C. Ghosh was the Director of the Indian Institute of Technology at Kharagpur. The vacancy for the post of a Professor and Head of the Department of Mathematics at the I.I.T. was advertised. A contemporary of Dr Ghosh (Dr B. Sen) approached him and said : "I wish to be an applicant for the post. But, if you happen to be a candidate, I don't have any chance whatsoever." Smilingly, Dr S. Ghosh assured his colleague that he would not stand in his way. He said, "I was at first inclined to apply for the post, because I felt that this opportunity was a gateway towards the fulfilment of a long-cherished dream to establish a centre for advanced studies in Mathematics in our country. But, on second thoughts, I felt that the administrative burden associated with the post of Head of Department may not prove conducive towards the attainment of my goal. Furthermore, the age for retirement at the I.I.T. is rather too short for a man of my age."

Somehow, this matter reached the ears of Sir J. C. Ghosh, to whom the name of Dr S. Ghosh had been strongly recommended by Professor S. N. Bose He requested Dr Ghosh through Professor N. R. Sen, to apply for the post. He also assured him that there would be provision for an assistant to lighten his administrative load and there was every likelihood of his services being extended beyond the prescribed retirement period. Dr Ghosh pondered over the matter for a while and then sent his reply expressing his inability to apply because he had previously given his word to a friend that he would not be a candidate. He was then informed that the authorities did not consider his friend to be the most suitable candidate and again requested him to apply. But the votary of truth could not be dislodged from his self-chosen path of righteousness. Eventually, the post went to Professor B. R. Seth.

Life is a series of disenchantments, and Dr Ghosh had to have his share of them. In 1958, he had a minor difference of opinion with the Head of his Department regarding the curriculum and distribution of teaching load among the members of the staff. As a result, he sent his letter of resignation. Fortunately, the university authorities ignored his letter because he was regarded indispensable to the Department of Mathematics. Subsequently, Dr Satish Chandra Ghosh, treasurer, Calcutta University, and a well-wisher persuaded him to join the Department of Pure Mathematics and soon he was elevated to the post of the Head of the Department.

After the retirement of Professor N. R. Sen in 1959, the post of Sir Rash Behari Ghosh Professor in the Department of Applied Mathematics fell vacant. It was offered to Dr Suddhodan Ghosh, although he was not a candidate for the same. He accepted the post with his customary humility. Thus, Dr Ghosh had the



unique distinction of becoming the Head of the Departments, of both Pure and Applied Mathematics, a rare and unique privilege.

Professor Suddhodan Ghosh retired from active service in the Calcutta University in 1963. His students, friends and admirers tried to persuade him to apply for the post of Scientist—Emeritus, subsidised by the University Grants Commission. But he stuck to his idealism of 'Non-seeking of any favour from any source', and turned a deaf ear to their requests.

## Role of a Teacher

Dr Suddhodan Ghosh was a born scholar and teacher. Early in his life, he vowed to dedicate his life to the pursuit and dissemination of knowledge. He kept this resolution to the last. During his post-graduate years, he occupied a single room in the outhouse of the family residence. It was his study-cum-bedroom. 'Early to bed and early to rise' was his motto. He would get up at 4 a.m., finish his morning ablutions, prepare a cup of coffee and sit among books. The family barber attended the household twice a week. His clients were many. And so, in order to save time for study, he preferred to sport long hair and a beard. They remained his hall-mark throughout life.

Clad in immaculate khadi, tall and handsome in appearance, with a flowing beard, he looked like an Aryan sage and captivated the hearts of his students. As a teacher, the methods adopted by him were often novel. Every year, he used to pick up new materials from different books and incorporate them in his lectures. He used to change his topics every two/three years for the benefit of his students and to keep himself up-to-date in every field of the teaching curriculum. Before taking up a new topic, he used to collect all available books on the subject from the university library and the local bookshops and study them thoroughly. Then he would scrutinise every aspect of the subject in detail, simplify the difficult points, work out the suggested problems and prepare extensive personal notes. He used to deliver his lectures extempore from his own notes, making his exposition easy and lucid for his students. What he taught was not really separable from his The material which he offered was not objective stuff; it was material personality. composed not only out of his mind and intellect but with his will and character. The students were spellbound and listened to his lectures with rapt attention. He did not resent recalling the salient points if any student failed to grasp the idea. He had to spend more lecture hours than usual for the benefit of his students. Before the university examinations, he helped the students by taking extra coaching classes on any topic suggested by them. As a matter of fact, he really enjoyed teaching even as an artist enjoys painting, a musician enjoys music or a sprinter enjoys competition on the racing track. Apparently brief and terse in his conversation with his students, he entertained a warm feeling of love and affection The silent flow of his affection could be felt by all and remembered for them. with deep gratitude.

As a research guide, he had hardly an equal. Assuming the role of an ideal Guru, he would teach quietly the relevant topics, day after day. He would help focus attention of his research associates on the essential points, generate a sense of



self-confidence in them and prepare them to tackle original problems. He looked into the future and kindled their creative faculties.

His punctuality and regularity were proverbial. During his long career of thirty-five years as a teacher in the Calcutta University, it is believed he availed himself of casual leave for only thirteen days. He never took any privilege leave.

# **RESEARCH ACTIVITIES AND CONTRIBUTIONS TO NEW KNOWLEDGE**

An ardent student of the late Professor S. N. Bose, Suddhodan Ghosh was irresistibly drawn towards Mathematical Theory of Elasticity. He developed a research interest in the subject even as a student of the Post-graduate class and published his first paper before joining the Department of Mixed Mathematics as Ghosh Research Scholar.

Subsequently, as lecturer in the same department, he worked mostly alone. There was hardly any provision for a salaried research scholar, associated with a lecturer at the Calcutta University. As his research activities widened, encompassing fields of both Applied and Pure Mathematics, many research associates, mostly honorary, congregated around him. He guided them in their doctoral theses. However, he never wrote a paper jontly with any student working under him for a doctoral degree. This was indeed a rare quality.

Ghosh's work comprises essentially two fields : Fluid Dynamics and Problems of Elasticity. The latter includes bending of plates, plane strain, plane stress, generalised plane stress problems in isotropic and aeolotropic bodies, vibration of ring, and torsion-flexure problems. An analytical survey of his scientific contribution is enumerated below :

# Fluid Dynamics

Liquid motion inside certain rotating circular arcs, with specified boundaries was discussed by Ghosh (1924), by solving Laplace's equation in  $\Psi$ , the stream function and its value on the boundary.

The problem, 'On the steady motion of a viscous liquid due to translational velocity of a tore parallel to its axis', was solved by him (1927) by using curvilinear coordinates  $(\alpha, \beta)$  and by assuming the Stoke's function  $\Psi = (\mu - \cos \beta)^{-3/2} \chi$ , where  $\chi$  denotes an infinite series of  $\cos n\beta$  and  $\alpha + 2\beta = \log\{(\rho + \alpha + iz)/(\rho - \alpha + iz)\}, (z, \rho)$ being cylindrical co-ordinates, the boundary conditions of  $\Psi$  having been expressed

as 
$$\Psi + \frac{1}{2} \rho^2 v = 0$$
 and  $\frac{\partial}{\partial n} (\Psi + \frac{1}{2} \rho^2 v) = 0$ .

Later, he persuaded Srimati Lakshmi Sanyal, a pupil and research associate, to examine critically the foundations of the three dimensional boundary layer theory on a surface. The asymptotic equations of the boundary layers were deduced in the same manner as that by Schmidt and Schröder (1942) in their determination of the equations of the two dimensional boundary layer theory. But Sanyal's (1957) equation contains a correction term which does not appear in Howarth's (1951) equation, and it has a form more precise than that of Howarth. With the

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help of Tensor Calculus, Sanyal also obtained boundary layer equations analogous to those of Lin and included a correction term, which was lacking in Lin's equation. Under the guidance of Ghosh, Sanyal published the following papers :—

- (i) On similarity solution of Prandtl's boundary layer equation (1955a).
- (ii) Two dimensional boundary layer flow along a wall in a converging channel with curved boundaries (1955b).
- (iii) Boundary layer in a converging nozzle with a spherical surface (1955c).
- (iv) The flow of viscous liquid in a circular tube under pressure gradients varying exponentially with time (1956a).
- (v) On the flow of a viscous liquid between two co-axial circular cylinders under a periodic pressure gradient (1956b).
- (vi) Equations of motion of an incompressible viscous fluid in parallel system of co-ordinates (1957a).
- (vii) Three dimensional boundary layer equations. (1957b).
- (viii) Jet issuing in all directions from a thin slit round a semicircular cylinder of small radius containing liquid under pressure (1957c).

# Elasticity

(a) Bending of Elastic Plates—In 1829, Poisson had solved the problem of bending of a loaded circular plate, where the load is uniformly distributed. Later, Saint Venant considered the case of the load being uniformly distributed round a concentric circle, while Michell (1902) took up the problem of a concentrated load at any point.

Ghosh (1925a) obtained the form of the deflection of a plate, when (i) the load is uniformly distributed between two concentric circles and two radii and (ii) the load is uniformly distributed along the arc of a concentric circle.

The solution of  $\nabla_1^4 \omega = \text{constant}$ , was obtained by him (1925b) in bipolar coordinates and employed to find the deflection of an elastic plate bounded by two non-concentric circles and bent by its own weight.

Happel (1921) gave the solution of bending of an elliptic plate loaded at the centre. Ghosh (1929) obtained the solution of a similar elliptic plate when the load is concentrated at the focus.

(b) Problems of Dislocation—This arises in a body occupying a multi-connected space when it is cut along a system of barriers put in to make the body simply-connected, the opposite faces being then joined by the removal or insertion of thin slices of matter of the same kind as that of the body. The new body so formed would then be in a state of initial stress. There is thus the physical possibility of the displacements in multi-connected space being expressed by many valued functions. Volterra (1907) had applied the solution of equations of equilibrium in polar co-ordinates to the problem of dislocation in a hollow cylinder bounded by two concentric circles. Ghosh (1926) used the expression for the stress function  $\chi$  in bipolar co-ordinates to solve the problem of dislocation in a hollow in a hollow cylinder whose cross-section is bounded by two non-concentric circles due to (i) parallel fissures and (ii) wedge-shaped fissures.



Ghosh (1930) worked out the solutions of biharmonic equation in elliptic coordinates and applied it to the problem of plane strain. A single-valued solution was found suitable for the case of an elliptic cylindrical cavity in an infinite solid, while another many valued solution applicable to the state of strain in an elliptic cylindrical shell, which had suffered dislocation due to a triangular axial fissure, was also found. Ghosh found that Love's method of solving problems in plane strain in elliptic co-ordinates is only applicable to cases where surface displacements are given. The method developed in Ghosh's paper gives not only Love's solution but also the solution of an infinite solid with elliptic cylindrical hollow subjected to a pressure on the inner elliptic boundary and solution of the problem of dislocation for a wedge-shaped fissure in a body bounded by two confocal elliptic cylinders.

(c) Plane Problems—The problem of determination of stress and strain in rotating elliptic cylinders and discs had been worked out by Chree (1892) by using cartesian coordinates. However, the expressions obtained by him are far from being simple. The method of solution of Basu and Sen Gupta (1927), by using elliptic coordinates, is only applicable to slowly rotating cylinders with nearly circular sections. Ghosh (1928) considered the problem as that of a plane stress (for cylinder) and a plane strain (for disc) by solving for the stress-function  $\chi$  from the equation  $\nabla_1^4 \omega$  = constant without any limitations.

The problem, "On the stress and strain in a rolling wheel", was worked out by Ghosh (1933) assuming the wheel to be in a state of generalised plane stress, and solving the stress equation of equilibrium. Solution of "Plane strain in an infinite plate with an elliptic hole," when the surface displacements are given was obtained by Ghosh (1936a) by using the form of the stress function in elliptic coordinates. "Stress distribution in a heavy circular disc held with its plane vertical by a peg at the centre" was discussed by Ghosh (1936c). He considered the disc to be in a state of generalised plane stress. Stress-function  $\chi$  is determined in two cases : when the peg is assumed (i) to be rigidly fixed to the disc so that it exerts a thrust on the disc across the upper half of the common boundary and tensions across the lower half and (ii) to be so inserted in the circular hole, so that the peg exerts tractions on the disc only across the upper half of the common boundary. Interesting cases of "Generalised plane stress in heavy circular disc," were considered by Ghosh (1936c). In the first case, considered in this paper, the stress-function suitable for heavy circular disc with a concentric circular hole resting vertically on a horizontal plane is determined. This applies to the case of a wheel to which a load is applied by an axle fitting into it, by prescribing suitable tractions on the inner boundary.

In 1937, an investigation was initiated by Ghosh for the solution of equations of generalised plane stress in a semi infinite plate with a singularity at a specified point arising from a couple acting on the plane of the plate.

The solution of the problem for a single circular hole in an infinite plane under tension is well-known, but it is seen that the presence of adjacent holes or boundaries complicates the situation. Bipolar co-ordinates were used by Ghosh (1939) to find "The stress distributions in an infinite plate containing two equal circular holes," when it is subjected to (i) a uniform tension in the direction of the line of centres, and (ii) a uniform shear in the plane of the plate.

Ghosh (1942) investigated the problem, "On plane strain and plane stress in aeolotropic bodies." Earlier, Haber (1938) had obtained the equation satisfied by the stress function assuming the aeolotropic plate to be in a state of generalised plane stress. Green and Taylor (1939) further simplified the equation assuming that the material of the plate had two directions of symmetry at right angles to each other in the plane of the plate. However, no justification had been given by the above authors for the simplifying assumptions made by them Ghosh (1942) examined the fundamental question whether an aeolotropic body could at all be in a state of plane strain or plane stress, and if so, under what conditions. Compatibility conditions play an important role in finding the necessary and sufficient conditions for plane strain. It was also observed that these conditions are satisfied in a large class of bodies and particularly in materials with three perpendicular planes of symmetry, one of which coincides with the plane of the plane strain. Ghosh also examined the case of plane strain for bodies with three perpendicular planes of symmetry at each point, one of them coinciding with the plane of plane stress and found that only possible stress functions in plane stress in the X-Y plane are polynomials of the sixth degree in X, Y. Such stress functions can only give rise to particular distributions of tractions on the rim of the aeolotropic plate so that the assumption of plane stress under an arbitrary distribution of surface tractions on the rim cannot be warranted.

Solutions are given of some simple problems of plane stress including that of the bending of a rectangular aeolotropic plate by couples applied to the rim. In continuation of the above paper, Ghosh (1943), found that a state of plane stress cannot in general, exist in the plate, if such a material with its plane faces parallel to one of the planes of symmetry be subjected to arbitrarily prescribed traction on its rim. He also solved some examples of plane stress that could occur in the plate requiring traction on the rim, to be distributed in some restricted manner. The distribution of stress in a rotating aeolotropic disc with a circular or elliptic boundary was also considered. It was found that the solution of the partial differential equation of the fourth order satisfied by the stress function which is not homogeneous leads to the strain function being a polynomial of the fourth degree in X, Y and quadratic in Z. It was also seen that the expression for the stress function satisfies the condition imposed for the existence of a plane stress in the plate. Ghosh's paper (1946c), "On Generalised plane stress" examines the possibility of a state of generalised plane stress in a plate of aeolotropic material with the planes of symmetry, the plane face of the plate being parallel to one of the planes of symmetry. The components of stress have been expressed in terms of two stress functions  $\chi$  and  $\psi$ . It is deduced that it satisfies two homogeneous linear partial differential equations of the sixth order which are not in general compatible unless it is a polynomial of a degree not higher than eighteen in X, Y, Z. It is thereby concluded that a state of generalised plane stress is, in general, not possible in such a plate.

Ghosh (1946a) critically examined the assumptions made in solving the problem of the plate being stretched and bent by forces acting along its cylindrical edge.



He found that in the flexure of a plate, a state of generalised plane stress is always possible while only a restrictive distribution of deforming forces and couples could lead to a state of plane stress. It was also pointed out that in the stretching of a plate by forces in its plane, a state of plane stress could always be found while the existence of a state of generalised plane stress has not been proved. He further found that the problem of average stresses can not be solved unless suitable assumption is made about  $Z_z$  which occurs in the expression for average stresses and average displacements. It was shown that the most general value of  $Z_z$  is a plane harmonic function, or the average value of the product of a harmonic function by X or Y or Z or  $\gamma^2$ .

Ghosh (1946a) also criticised that the solution of the problem of Filon (1903) by taking  $Z_z = 0$  identically and  $X_z = Y_z = 0$  on the boundary plane,  $Z = \pm h$ , leads to the vanishing of  $X_z$  and  $Y_z$  identically. Hence, Filon's solution is not of generalised plane stress, but one of plane stress.

Southwell (1936) gave a solution of stress equation of equilibrium which satisfies the conditions of compatibility of stress and the condition  $Z_z = 0$ . Ghosh pointed out that in the above solution, Southwell had not taken account of the condition that the plane faces of the plate are free from tractions.

In his paper entitled, "On the concept of generalised plane stress in an aeolotropic plate," Ghosh (1946c) gave an exact solution of stress equation of equilibrium satisfying the stress compatibility relations as well as boundary conditions on the plane faces of the plate, thus proving the possibility of a state of generalised plane stress in the plate. The appropriate surface tractions which are to be applied to the edge of the plate to maintain generalised plane stress were also calculated by him.

Ghosh in a paper (1944) concludes that an infinite elastic solid cannot be in a state of plane strain when forces are distributed uniformly perpendicular to several infinite parallel straight lines, the forces on each line being in a definite direction, unless the algebraic sums of the components of the forces per unit length of the lines along two perpendicular directions at right angles to them vanish. He also observed that when an infinite plate is deformed by forces in its plane acting at given points, the average stress and displacements in the plate can be obtained from the analogous plane strain problem provided the forces acting on the plate at the given points are either in equilibrium or reduce to a couple.

Solution of Laplace's equation suitable for the problem when the boundaries are two spheres touching each other was obtained by Ghosh (1936d).

(d) Vibration of Ring—Free vibrations of a rod which in the unstressed state forms a circular ring have been discussed by several writers after the 'rotating inertia' terms have been neglected from the equations of motion. Ghosh (1935b) examined the retention of these terms to the period of vibration and observed that as the number of wavelengths in the circumference increases, this correction increases in importance in the flexural vibrations while it is small and remains practically stationary for torsional vibrations.

Ghosh (1936b) gave certain solutions of equilibrium equations in three dimensions on the assumption that the displacement is inversely proportional to



 $r^{2}$  and determined the distribution of stress in a cone to which a couple is applied at the vertex.

(e) Torsion and Flexure Problem—Ghosh (1935a) gave a solution of the flexure problem for a beam whose cross-section consists of (1) a semi-ellipse bounded by its minor axis and (2) an ellipse and two confocal hyperbolas. The second one was reduced to the interesting case of an elliptic beam with two cracks extending the foci to the boundary of the ellipse along the major axis.

Torsion of a solid of revolution of a material possessing curvilinear aeolotropy was solved by Ghosh (1956).

Pioneering work had been done by Ghosh (1947, 1948) in solving torsion-flexure problems by the use of function theory. It has opened out the possibility of solution of a new class of boundaries.

The use of complex function had been made by Stevenson (1938-39). Morris (1940) had also used complex variables in solving some general types of sections. Ghosh observed that Stevenson's subdivision of the problem into six Dirichlet or Neumann problems, when the origin is not at the C. G. is defective in as much as the six flexure functions introduced by him are not free from infinities. In Morris's solution, the series she makes use of are not all convergent within the cross-section.

Solution of the torsion problem of isotropic elastic cylinder proposed by Muskhelisvili (1929) by function-theoretic method is of remarkable simplicity ann elegance. No attempt had yet been made to apply function-theoretic method to the solution of the flexure problem. The boundary condition in the flexure problem when formulated as a Dirichlet one is not simple. Ghosh (1947*a*) simplified the problem by subdividing the flexure function into three simpler Dirichlet functions  $f_0$ ,  $f_1$ ,  $f_2$  with boundary conditions of their imaginary parts  $\psi_0$ ,  $\psi_1$  and  $\psi_2$  as

$$\psi_0 = \frac{1}{2} (x^2 + y^2), \ \psi_1 = \int_A^p [(1 + \sigma) x^2 - \sigma y^2] dy + \text{const.}$$
  
and  $\psi_2 = -\int_A^p [(1 + \sigma) y^2 - \sigma x^2] dx + \text{const.}$ 

For such a sub-division to be possible, it is essential that the origin should be taken at the C. G. of the cross-section. The orientation of the load line in the terminal cross-section has been taken rather arbitrarily.

The boundary condition is-

$$\psi_2 + i\psi_1 = \frac{1}{12} (1 + 2\sigma) z^8 - \frac{1}{4} z z^2 + \frac{1}{2} (1 + \sigma) \int_A^{\sigma} z^2 dz$$

The analytic functions  $f_1$  and  $f_2$  are then expressed as :

$$f_1 = \frac{1}{12} (1 + 2\sigma) z^3 + f_{11} + (1 + \sigma) f_{12},$$
  
$$f_2 = \frac{1}{12} (1 + 2\sigma) iz^3 + f_{21} + (1 + \sigma) f_{22},$$

where  $f_{11}, f_{21}, f_{12}, f_{22}$  are obtained from the second and third term respectively of the expression for  $\psi_2 + i\psi_1$ .



The solution using  $z = \omega(\sigma)$ , the formula mapping the region conformally into the unit circle  $\gamma$  has been finally obtained by using Schwarz's formula as—

$$f_{0} = \frac{1}{2\pi} \int_{r} \frac{\omega(\sigma) \overline{\omega} (1/\sigma)}{\sigma - \rho} d\sigma$$

$$f_{11} + i f_{21} = \frac{i}{2\pi} \int_{r} \frac{[\omega(\sigma)]^{2} \overline{(\omega 1/\sigma)}}{\sigma - \rho} d\sigma$$

$$f_{11} - i f_{21} = \frac{i}{2\pi} \int_{r} \frac{\omega(\sigma) [\overline{\omega} (1/\sigma)]^{2}}{\sigma - \rho} d\sigma$$

$$f_{12} + i f_{22} = -\frac{i}{2\pi} \int_{r}^{\rho} d\rho \int_{r} \frac{[\omega(\sigma)]^{2} \omega'(1/\sigma)}{\sigma^{2} (\sigma - \rho)} d\sigma$$

$$f_{12} - i f_{22} = -\frac{i}{2\pi} \int_{r}^{\rho} d\rho \int_{r} \frac{[\overline{\omega} (1/\sigma)]^{2} \omega'(\sigma)}{\sigma - \rho} d\sigma$$

The coordinates of the centre of flexure had also been obtained by Ghosh. Utilising Ghosh's method (1947-48), Dr D. N. Mitra, the first research associate of Dr Ghosh, continued this line of investigation. Mitra (1948) solved the problems of torsion flexure for (i) inverse of an ellipse, (ii) two orthogonal circular arcs and (iii) loop of leminiscate of Bernoulli. Subsequently, he worked out the cases of the flexure of limacon (1949c) flexure of cross-section bounded by two closed curves (1952) and the torsion of composite sections (1955), in succession.

Development of the function-theoretic method reveals his remarkable mathematical ability, when Ghosh solves the torsion problem (1947b) and flexure problem for boundaries (1948a) which consist partly of as traight line by extending the theory propounded in his paper (1947a). In such cases, the function mapping the region conformally on the unit circle is very complicated while the function which maps the cross-section conformally on a semi-circular area is simple, the straight boundary of the cross-section corresponding to the boundary diameter of the circle. The problem is then reduced to the determination of a function of p analytic within the semi-circular area and real on the bounding diameter, its imaginary part taking up prescribed values on the semi-circumference. By Schwarz's principle of reflection, this function is continued analytically to the other half of the unit circle so that the problem becomes one of determination of a function analytic within the unit circle and its imaginary part takes up prescribed values, on the circumference. This function is then determined with the help of Schwarz's The method has been extended by Ghosh (1948b) to "Torsion and formula. flexure of a beam whose cross-section is a quadrant of a given area," symmetrical about two particular axes and which can be mapped conformally on the unit circle by a function of a simple form. The solution of torsion-flexure problem depends



in this case on the determination of a function analytic in the region and whose imaginary part takes up prescribed values at the boundary. This function when continued analytically to other three quadrants of the circle R by Schwarz', principle of reflection is found to be multi-valued in the domain with a branch point at the point of intersection of the axes of symmetry. The part of the function which is multivalued is obtained from the value of the imaginary part of the function on the boundary of the region, and the remaining part of the function is then determined by representing R conformally on the unit circle and using Schwarz's formula for an analytic function within the circle, whose imaginary part takes up given values on the circumference. Using this method, Mitra has solved the torsion flexure of a semi-circular cylinder (1949a), semi-cardoid (1949b) sector of a circle (1950) and sector of a curve (1951). Bassali (1961) has also used Ghosh's method to solve "torsion of elastic cylinder with regular curvilinear cross-section."

Professor Suddhodan Ghosh made significant contributions to Mathematics and Mechanics. In particular, he was one of the leading research workers in the fields of Elasticity and Fluid dynamics in our country. Many of his papers have been quoted in standard text-books on Elasticity and relevant books of reference. The well-known book entitled, "Resistance des Materiaux" by Robert L' Hermite, I, Paris (1954) has treated the problems of Torsion and Flexure as solved by Ghosh and Mitra in great detail.

Among his many research associates, some persons distinguished themselves by following his footsteps like : (1) Professor Debendranath Mitra, Ex-Head of the Department of Mathematics, Indian Institute of Technology, Kharagpur; (2) Late Professor B. Karunes, Ex-Head of the Department of Applied Mechanics, Indian Institute of Technology, New Delhi; (3) Shri Kamini Kumar Dey, (Retd.) Assistant Professor of Mathematics, Presidency College, Calcutta; (4) Dr Jnangopal Chakrabarti, Professor, Department of Applied Mathematics, Calcutta University, Calcutta; (5) Dr Srimati Lakshmi Sanyal, Principal, Sarojini Naidu Girls' College, Calcutta; and (6) Professor Sudhir Khamrui, Reader, Department of Mathematics, Jadavpur University, Calcutta.

## REMINISCENCES OF FAMILY AND PRIVATE LIFE

Suddhodan Ghosh remained a bachelor for life. He rigidly adhered to the vow of celibacy of the cloister.

After his mother's demise in 1919, a rift appeared within the family. As a consequence he decided to leave his home and sever his ties with the family once for all.

For the rest of his life, he lived alone and aloof from the vagaries of the world, occupying a single-room flat, which he converted into a study-cum-bedroom. His only furniture consisted of three almirahs, a couple of chairs and a wide wooden bedstead piled with books and papers. He seldom used the ceiling fan lest the loose sheets of paper should fly away. He prepared his own meals which consisted of boiled rice and vegetables. Later, he further simplified his food-habits by resorting to milk and bread. He washed his own clothes and utensils and performed



all household chores. Only on rare occasions, during illness, did he engage a servant.

During the long vacations, he visited most of the places of pilgrimage in India. Jokingly, he used to say that by such visits he had assured for himself, a niche in heaven.

When he earned his first scholarship, he promised that he would give away five times the total amount of earned scholarships to charities. He kept his promise. Anonymously, he gave away about a lakh of Rupees to religious, social and charitable organisations like Sri Ramakrishna Mission, Bharat Sevasram Sangha, Mother Teresa's Sisters of Charity, Salvation Army, Red Cross Society etc. He also donated liberally when natural calamities ravaged the country. It should be noted that he did not belong to the class of men who gave away donations merely for recognition. He was one of those who had little and gave it all away. He was a silent donor. He used to say: "Monotheism is my doctrine. By offering a helping hand, I really help myself." After giving about a thousand Rupees to a needy friend, he whispered aside, "I guess I shall be able to sleep more comfortably tonight." He used to give monthly stipends to needy meritorious students and poor freedom-fighters.

About a week before his death, all the gold medals he had obtained during his brilliant academic career were handed over to the University of Calcutta in deference to his wishes, for the creation of an endowment for providing financial aid to poor and meritorious students of the Department of Applied Mathematics. He also bequeathed to the Asutosh Museum of the University of Calcutta, the valuable Gold Mohur of Akbar's time, he had received from his mother and had retained as a memento.

During a pilgrimage to the Himalayas he had visited the temples of Badrinarayan and Kedarnath. He prostrated himself on the icy floor before Lord Siva at Kedarnath and sought his benediction with the following words : "Father ! Bestow on me enough strength to stick to my resolution, once it is approved by my conscience, and endow me with the faculty of non-attachment to any mundane favour." Verily, he never bent his head before social injustice and stuck to his principles to the very end.

Prosperity never turned his head nor did adversity ever crush his spirit. He took the joys and sorrows of life with imperturbable calm. He had no hankering for pleasant things, nor hatred for evil ones. He did not expect anything, nor did he grieve. He took things as they came, and pursued his path of life with steady, unfaltering steps. Although, everybody was welcome to him for his advice, he never visited any house for social contact. In this sense, he was never social. He was above society or social customs.

In his academic life, he behaved like a true Sanyasi, never requiring more than was needed for his maintenance. He refused to accept head examinerships of the university.

He felt that self-satisfaction was the surest sign of degeneration. He rendered inestimable service to the Calcutta Mathematical Society, and helped to sustain it when it was on the verge of collapse. For a long time, he functioned as its (honorary) Editorial Secretary. Although in later years, he cut off his relations with the society, he was elected its honorary member, a unique distinction.



## HONOUR

He was elected a Fellow of the National Institute of Sciences of India in 1951.

### EPILOGUE

Professor Suddhodan Ghosh retired from the Calcutta University in 1962. He did not seek any extension to his services. He had walked through the politics of his time, touching elbows with his contemporaries as he met and passed them on his pilgrimage through the Department of Applied Mathematics. He knew the obvious realities of his environment.

There was no provision for pension to a retired university teacher. Nevertheless, he got about Rs. 93,000 as his provident fund and gratuity. To Dr Ghosh money was no lure—it was simply something to give away. He rented a single-room flat on the ground floor of 256, Rash Behari Avenue at Rs. 75 per month and reduced his personal expenses to the minimum. An asthmatic afflicted by a weak heart, his life expectancy was poor. And so, he began supporting destitute friends and poor students rather lavishly. His accumulated wealth dwindled rapidly. At this critical juncture, a friend came to his rescue. He withdrew the remaining money from the bank and kept it under his personal custody as a trustee. He gave Dr Ghosh a monthly allowance of Rs. 300 only for his personal expenses.

During the initial years after his retirement, Dr Ghosh was agile and vivacious. He took long walks into the country, enjoying its scenic beauty. His interest in literature was deep. Tagore was his favourite author. He also enjoyed reading books of Tolstoy, Goethe, Victor Hugo, Dickens and George Bernard Shaw.

With declining health, his life became slower, more austere and introspective. As his eyesight progressively deteriorated, he confined his activities within the four walls of his room. A friend suggested the surgical removal of cataracts from his eyes. He refused. His reply was characteristic. He said, "All my life I have seen and enjoyed the beauty of the external world. At its fag end, I am interested in exploring the inner world. For this purpose, physical eyesight is not necessary."

He suffered in silence, but never expressed his anguish in words. To any casual enquirer about his health, his usual answer was : "I am extremely well. God has taken special care of me." His concept of Providence is revealed in the following tete-a-tete which he once had with his philosopher friend, Dr Tripurari Chakravarti, a frequent visitor.

- Dr C : "Dr Ghosh, do you really believe that God exists?"
- Dr G : "Yes, I do."
- Dr C : "How do you know? Have you ever seen Him?"
- Dr G : "No, I have not seen Him visually. And yet, I have felt His existence in my heart. It is not necessary to visualise everything to believe in its existence. I have never visited England. Does that mean that England never exists?

The ancient Indian sages emphatically maintained that they had realised God within themselves. How can I disbelieve them?"



### **Biographical Memoirs**

And then, with a twinkle in his eyes, he smilingly added : "Perhaps His seat is spread in the hearts of a few saints, even as one finds singularities in the analytic regions of Mathematics."

He loved, respected and adored Professor S. N. Bose. The adoration was mutual. On hearing of the demise of Professor Bose, from the present writer, his sightless eyes glistened with tears. He earnestly requested the author to get him a pair of old slippers of Professor Bose. On being asked what he proposed to do with them, he said, "I shall put them on my head every morning."

During the evening of his life, with the dark clouds of uncertainty hanging over his mental horizon, Dr Ghosh realised two central facts of life : the futility of our efforts, and the tender and loving watchfulness of Providence. During this difficult period when he was practically bedridden, he was fortunate to secure the services of two of his erstwhile pupils, Dr Lakshmi Sanyal, M.A., D.Phil. and Dr Mina Majumder M.Sc., Ph.D. They helped him through crisis and peril and in moments of despair. With genuine affection and unpretentious piety, they adopted the role of little mothers to nurse this great man and kept close company with him till his death.

About a week before his passing away, he gave away all his earthly possessions; books, papers, furniture money and gold medals. Earlier, he had given away anonymously about a lakh of rupees to different social and charitable organisations. Now he distributed his books and papers among friends and pupils. In particular, he presented O. W. Richardson's Electron theory of Matter to the present writer, who was amazed to find the signature of Ernest Rutherford, 1919, on its cover slip. It is amazing how the book found its way into the Calcutta second-hand book market !

He died as he had lived—bravely and quietly on May 6, 1976. His last rites were performed by Bharat Sevasram Sangha. To his two beloved pupils, to his friends who knew and loved him and to numerous admirers who sorrowed at his death, he left behind the shining legacy of courage, righteousness and profound scholarship.

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S. D. CHATTERJEE

#### REFERENCES

BASU, N. M., and SEN GUPTA, H. M. (1927) Bull. Cal. math. Soc., 18, 141.
BASSALI, W. A. (1961) Proc. Camb. phil. Soc.
FILON, L. N. G. (1903) Phil. Trans. R. Soc., A. 201, 63.
GREEN, A. E., and TAYLOR, G. I. (1939) Proc. R. Soc. London, 173A, 162.
HAPPEL (1921) Math. Zeit., 11, 194.



#### Suddhodan Ghosh

HOWARTH, L. (1951) Phil. Mag., 42, 239-243.

HUBER, M. T. (1938) Contributions to the Mechanics of Solids, P. 80.

MICHELL (1902) Proc. London math. Soc., 34, 223.

MITRA, D. N. (1948) Bull. Cal. Math. Soc., 40, 4, 173.

- (1949a) Ibid, 41, 1, 28.

- (1949b) Ibid, 41, 125.

- (1949c) Ibid, 41, 153.

- (1950) Ibid, 42, 3, 131.
- -(1951) Ibid, 43, 1, 41.
- (1952) Ibid, 44, 4, 143.
- (1955) Ibid, 47, 3, 191.

MORRIS, R. M. (1940) Proc. London math. Soc., 46.

MUSKHELISVILI, N. I. (1929) Atti dille reale acad. natn. dei Lincei, 6, 9, 295.

POISSON (1829) Mem. Paris. Acad., t. s.

SANYAL, L. (1955a) Proc. natn. Inst. Sci. India, 21A, 8-14.

- (1955b) Bull. Cal. math. Sc., 47, 3.

- (1955c) Ibid, 47, 65-70.

- (1956a) Indian J. Phys., XXX, 57-61.

- (1956b) J. Tech., 1, 1.

- (1957a) Indian J. Phys. XXXI, 6-10.

- (1957b) Zeit. f. ang. math. U. Mech., 5/6, 169-177.

- (1957c)Zeit. f. ang. Math. U. Phys. VIII, 156-159.

SCHMIDT, H., and SCHRÖDER, K. (1942) Luft fahr Farschung, 19, 65-97.

SOUTHWELL, R. V. (1936) Phil. Mag., 21, 201.

STEVENSON, A. C. (1938-39) Phil. Trans. R. Soc. London, A 257.

VOLTERRA (1907) Paris. Ann. Worm., 24, 40.

#### **BIBLIOGRAPHY**

1924. On liquid motion inside certain rotating circular arcs. Bull. Cal. math. Soc., 15, 27-46.

1925a. On a problem of elastic circular plates. Bull. Cal. math. Soc., 16, 63-70.

- -b. On the solution of  $\Delta_1^4 \omega = C$  in bipolar coordinates and its application to a problem in elasticity. Bull. Cal. math. Soc., 16, 117-122.
- 1926. On certain many valued solutions of the equations of elastic equilibrium and their application to the problem of dislocation in bodies with circular boundaries. Bull. Cal. math. Soc., 17, 185-194.
- 1927. On the steady motion of viscous liquid due to translation of a tore parallel to its axis. Bull. Cal. math. Soc., 8, 185.
- 1928. On plane strain and stress in rotating elliptic cylinders and discs. Bull. Cal. math. Soc., 19, 117-126.
- 1929. On the bending of a loaded elliptic plate. Bull. Cal. math. Soc., 21, 191-194.
- 1930. On the solution of the equations of elastic equilibrium suitable for elliptic boundaries. Trans. Am. math. Soc., 32, 47.
- 1933. On the stress and strain in a rolling wheel. Bull. Cal. math. Soc., 25, 99-106.
- 1935a. Flexure of beams of certain forms of cross-sections. Bull. Cal. math. Soc., 27, 61-68.
- -b. A note on the vibrations of a circular ring. Bull. Cal. math. Soc., 27, 177-182.
- 1936a. Plane strain in an infinite plate with an elliptic hole. Bull. Cal. math. Soc., 28, 21-47.
  -b. One some simple distributions of stress in three dimensions. Bull. Cal. math. Soc., 28, 107-119.
  - -c. Stress distribution in a heavy circular disc held with its plane vertical by a peg at the centre. Bull. Cal. math. Soc., 28, 145-150.
  - -d. On the solutions of Laplace's equation suitable for problems relating to two spheres touching each other. Bull. Cal. math. Soc., 28, 193-198.



-e. On some two dimensional problems of elasticity. Bull. Cal. math. Soc., 28, 213-222.

- 1937. On the distribution of stress in a semi-infinite plate under the action of a couple at a point in it. Bull. Cal. math. Soc., 29, 177-184.
- 1939. Stress distribution in an infinite plate containing two equal circular holes. Bull. Cal. math. Soc., 31, 149-159.
- 1942. On plane strain and plane stress in aeolotropic bodies. Bull. Cal. math. Soc., 34, 157-169.
- 1943. Stress systems in rotating aeolotropic discs. Bull. Cal. math. Soc., 35, 61-65.
- 1944. On the divergence of the solution of a problem of plane strain. Bull. Cal. math. Soc., 36, 51-58.
- 1946a. A note on average stresses in a plate. Bull. Cal. math. Soc., 38, 10-20.
  - -b. On the concept of generalized plane stress. Bull. Cal. math. Soc., 38, 45-56.
- -c. On generalized plane stress in an aeolotropic plate. Bull. Cal. math. Soc., 38, 61-66.
- 1947a. On the flexure of an isotropic elastic cylinder. Bull. Cal. math. Soc., 39, 1-14.
- -b. On a new function—theoretic method of solving the torsion problem for some boundaries. Bull. Cal. math. Soc., 39, 107-112.
- 1948a. On the flexure of a beam whose cross-section is bounded partly by a straight line. Bull. Cal. math. Soc., 40, 77-82.
  - -b. On the torsion and flexure of a beam whose cross-section is a quadrant of a given area. Bull. Cal. math. Soc., 40, 107-115.
- 1956. Torsion of a solid of revolution of a material possessing curilinear aeolotropy. J. Assoc. appl. Phys. Cal. Univ., 3, 1-4.