C. T. Rajagopal.

CADAMBATHUR TIRUVENKATACHARLU RAJAGOPAL

(1903–1978)

Elected F.N.I. 1964

MANY who knew C. T. Rajagopal could not believe that he was no more on April 25, 1978. He was so healthy and was so regular in habits that nobody could believe that destiny would snatch him away so soon. The end was painless and his wife who was by his side could not believe that he had left her. A medical man, who was called by her, when her beloved husband stopped responding to her enquiries, declared him to have succumbed to a "massive heart-attack."

BIRTH AND EDUCATION

Rajagopal was born to Cadambathur Tiruvenkatacharlu and Padmamma, on September 8, 1903. Thiruvenkatacharlu, as his name indicates, spent most of his life in the Telugu-speaking areas of composite Madras State of those days, serving the Revenue Department with quasi-judicial duties. Rajagopal was the eldest son, with two younger brothers and a sister of whom one brother and the sister survived him. His aged mother, who survived Rajagopal breathed her last a few months later. Rajagopal and his brother Venugopal, whose official names were, however Raju and Venu, respectively, were well known in Triplicane for their high degree of proficiency first as students of the Hindu High School and later, of the Madras Presidency College. He matriculated in the year 1919 and passed the Intermediate Examination of the University of Madras in 1921, both with high marks. He took the B.A. (Honours) Course in mathematics the same year but postponed writing the final examination by one year because of illness. He passed the B.A. (Honours) Examination obtaining the first rank in the then Madras Presidency in the year 1925. It was during the B.A. (Honours) course that he came into contact with K. Ananda Rau, who was Professor of Mathematics in the Presidency College. Clearly, Ananda Rau made an indelible impression on Rajagopal, which lasted throughout his life. In fact, there is an interesting parallel between the careers of these two men, which begins even earlier than their first contact, viz., their highly proficient studentship in the two renowned institutions, the Hindu High School and the Madras Presidency College. Ananda Rau could notice the "exceptional intelligence" of young Rajagopal. It is noteworthy that T. Vijayaraghavan, who influenced Rajagopal later in his career, was a senior college mate of his in the Honours classes.

Professional Career

Rajagopal worked for more than a year in the Madras Accountant General's Office but left it to begin his teaching career in the Annamalai University in 1930–31. He later moved to the Madras Christian College and served that college for two decades till 1951. It is here that his lucid and often original exposition of classical mathematical analysis and his tender care for students, in which he was in full measure supported by his wife, Rukmini, whom he married in the year 1925, won for him their everlasting admiration, affection and respect. He built a house at Tambaram according to his own design and settled down, as he thought at that time, and named his abode "The Anchorage," teaching mathematics and doing research. Unlike many of Ananda Rau's students, Rajagopal started his research work late, some 14 years after his graduation. His initial papers were a prelude to what he was to take up for research and reflected his original exposition of topics which he taught for the honours classes.

At the Ramanujan Institute of Mathematics, Madras

The year 1951 was really a turning point in Rajagopal's life. By then Vijayaraghavan had taken up the headship of the Ramanujan Institute of Mathematics founded in 1950 by the late Alagappa Chettiar,* a well-known educationist. Vijayaraghavan succeeded in persuading Rajagopal to join the Institute. Rajagopal yielded readily to the persuasion of his great friend. However, the sudden and unexpected demise of Vijayaraghavan in 1955 left Rajagopal in sole charge of the responsibility of keeping alive "a small remembrance of a great man" as Alagappa Chettiar described the Ramanujan Institute. Thereafter, Rajagopal's life was a reflection of the vicissitudes in the history of the Ramanujan Institute. Professor Rajagopal was well known by this time and was affectionally called CTR by his students and as Raju by those who were close to him.

After Vijayaraghavan's demise CTR was made Director of the Ramanujan Institute of Mathematics by Dr Chettiar on the advice of Professor Andre Weil and on the opinion of eminent mathematicians like R. P. Boas (Jr.) and A. Zygmund. Later, when the fate of the Institute hung on the balance, the intervention by the famous astrophysicist Professor S. Chandrasekhar made the then Prime Minister Jawaharlal Nehru make a recurring grant to the Institute and transfer its management to the University of Madras. The Institute retained its separate identity till 1967 when it became enlarged, with the University Department of Mathematics merging with it, to become the Ramanujan Institute for Advanced Study in Mathematics. Rajagopal's contribution to the Institute attaining the supreme position as one of the most important centres of mathematical research in India was immeasurable. Professor Rajagopal who was due to retire in 1965 on

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*Sir Rm. Alagappa Chettiar, Industrialist and Philanthropist was responsible for the development of Karaikudi as a major educational Centre. It was his offer of 500 acres of land from his Estate and a grant of Rs. 15 lakhs which resulted in the establishment of the Central Electro-Chemical Research Institute at Karaikudi.

superannuation was given an extension of service and continued to be head of the enlarged Institute. It should be mentioned that before all this was achieved, the Institute passed through many a crisis and was often threatened with closure. That is survived those gloomy and agonizing days was as much due to the high quality of mathematical activity generated by CTR as it was to the indefatigable persistence, faith and hope with which he went on representing its cause to the authorities. He finally retired from the Institute in June 1969 but continued his research work with a small financial assistance through the Institute. It is no exaggeration to say that the Institute, as it stands today, is a monument to the unremitting efforts of Rajagopal as it is to the memory of Srinivasa Ramanujan. It is also true that the Ramanujan Institute is finding it difficult to face, with courage, the assessment of its achievements because of the void created by CTR's retirement. The research work of the Institute was during Rajagopal's time, so productive and significant that his passing away has created a vaccum in the country among active, hard analysts, of whom he was perhaps the last.

CONTRIBUTION TO MATHEMATICS

The classical branches in every field of research are always the more difficult to work in, since original ideas are scarce. Rajagopal's main fields of research were summability methods and complex function theory, specially analytic and entire functions.

That in these classical fields CTR acquired an international reputation speaks for the depth, originality and high quality of his work. A few of his earlier papers were an outcome of his pedagogical innovation. Besides the main fields of his interest mentioned above, in these earlier years he was inspired by an article of C. M. Whish, an officer in the employ of the British East India Company of those days, dating back to 1835. This paper pointed out that the series for sine, cosine and arc tangent, as also better and better approximations for π, are given in certain astronomical texts available in Kerala. Rajagopal took upon himself the task of establishing, to the satisfaction of mathematicians and mathematical historians, that the fundamentals of mathematical analysis were discovered and used in Kerala at least two centuries before Newton and Leibniz. He was assisted in this task by the late Prince Rama Varma of Cochin and his own students.

CTR's papers thus fall under three heads. (i) sequences, series, summability, (ii) Functions of a Complex variable, (iii) History of medieval Kerala mathematics. It is not an easy task to summarize here the contents of all of his numerous papers. What follows is an account of that part of CTR's work which he himself considered, in his own modest opinion, interesting and representative of his work.

Sequences, Series and Summability

Rajagopal's work in this field is well indicated in the only treatise on the topic by G. H. Hardy (*Divergent Series,* Oxford, 1949, especially in the notes to § 7.7), in the compendium by K. Zeller (revised later with the collaboration of

W. Beekmann) (*Limitierungsverfahren,* Springer, 1Edn. 1958, 2Edn. 1970) and in the monograph entitled *"Typical Means"* by K. Chandrasekharan and S. Minakshisundaram (Published by the Tata Institute of Fundamental Research Bombay : Oxford, 1952). About a dozen of his papers related to the generalization and/or unification of the several tests for convergence for series of positive terms—without explicit connexion with summability. These provided the undercurrent for the theme of generalization and unification of Tauberian theorems which was to be the main theme of Rajagopal. CTR began his work in summability methods in a note entitled : on 'converse theorems of summability' with the plea that his note "did not merit attention of workers in the field of summability." That this was an expression of Rajagopal's characteristic modesty** is revealed by the attention which G. H. Hardy directed to it in his favourite (according to his collaborator J. E. Littlewood) treatise (*loc. cit.*). It is worthwhile tracing the history of this first research paper of CTR's.

It is well known [*see* Hardy, *loc. cit.*, 121, Theorem 64] that if (s_n) is a real sequence,

$$\frac{s_0 + s_1 + \ldots + s_n}{n+1} \to s,\ s_{n-1} = O_L\left(\frac{1}{n}\right),\ n \to \infty \Rightarrow s_n \to s,\ n \to \infty,$$

or, in other words, if $\{s_n\}$, $s_n = a_0 + \ldots + a_n$, $n = 0,1,2, \ldots$, is the sequence of partial sums of the series $\sum_{n=0}^{\infty} a_n$, then

$$\Sigma\, a_n = s\ (C,\ 1),\ a_n = O_L\left(\frac{1}{n}\right),\ n \to \infty \Rightarrow \Sigma\, a_n = s.$$

With reference to a strictly increasing positive sequence $\{\lambda_n\}$ diverging to infinity one can raise the question : Do we have

$$(\alpha)\quad \lim_{n\to\infty} \frac{\sum_{r=0}^{n} (\lambda_{r+1} - \lambda_r)\, s_r}{\lambda_{n+1}} = s,\ a_n = O_L\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right) n \to \infty \Rightarrow \Sigma\, a_n = s\,?$$

The first hypothesis on the left side of (α) is what is familiarly known as discrete Riesz summability of type λ_n and order 1 of $\Sigma\, a_n$ and denoted otherwise by $\Sigma\, a_n = s(R,\ \lambda,\ 1)$. CTR's result can be stated as

$$(\beta)\quad \Sigma\, a_n = s(R, \lambda, 1),\ a_n = O_L\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right),\ n \to \infty \Rightarrow s = \frac{\nabla \lim s_n.}{n \Rightarrow \infty}$$

This result supplements the negative answers to (α) given by K. Ananda Rau [*Proc. London math. Soc.*, (2), **30**(1930), 367–372].

Typical Abel summability (A, λ) is defined by :

$$\Sigma\, a_n = s\ (A, \lambda) \Leftrightarrow \sum_{n=0}^{\infty} a_n\, e^{-\lambda_n y} \text{ converges for } 0 < y < \delta$$

$$\text{and}\quad S\,(y) = \sum_{n=0}^{\infty} a_n\, e^{-\lambda_n y} \to s \text{ as } y \to +0.$$

**He used to describe himself in official records as the author of some 'thirty independent papers.'

Ananda Rau (*loc. cit.*) proved that

$$\Sigma\, a_n = s\,(A, \lambda),\ \lambda_{n-1} \sim \lambda_n,\ a_n = O_L\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right) \Rightarrow \Sigma a_n = s$$

and he showed that the hypothesis $\lambda_{n-1} \sim \lambda_n$ cannot be dispensed with. CTR established, using a result of Szász [*Trans. Am. math. Soc.*, **39** (1935), 117–130], viz.,

$$\Sigma a_n = s\,(A, \lambda),\ \sum_{r=0}^{n} a_r\, \lambda_r = O_L\,(\lambda_n) \Rightarrow \Sigma\, a_n = s\,(R, \lambda, 1)$$

that the hypotheses on the left of (α) are to be strengthened to obtain an affirmative answer. The following results give the stronger hypotheses required exhaustively and simultaneously for summabilities (A, λ) and $(R, \lambda, 1)$.

(i) $\displaystyle\sum_{n=0}^{\infty} a_n = s\,(A, \lambda)$ or $(R, \lambda, 1)$, $a_n = O\left(\dfrac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right)$, $n \to \infty \Rightarrow \Sigma a_n = s$

(ii) $\displaystyle\sum_{n=0}^{\infty} a_n = s\,(A, \lambda)$ or $(R, \lambda, 1)$, $a_n = O_L\left(\dfrac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right)$,

$$\lim \frac{\lambda_{n+1}}{\lambda_n} = 1,\ n \to \infty \Rightarrow \Sigma\, a_n = s.$$

(iii) $\displaystyle\sum_{n=0}^{\infty} a_n = s\,(A, \lambda)$ or $(R, \lambda, 1)$, $a_n = O_L\left(\dfrac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right)$,

$$\lim\ a_n \geqslant 0,\ n \to \infty \Rightarrow \Sigma a_n = s.$$

(iv) $\displaystyle\sum_{n=0}^{\infty} a_n = s\,(A, \lambda)$ or $(R, \lambda, 1)$, $a_n = O_L\left(\dfrac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right)$,

$$a_{n-1} = O_L\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right) \Rightarrow \Sigma a_n = s.$$

This sort of exhaustive study of typical summabilities is characteristic of CTR's work as, for instance, his results on the relation of limitation theorems to high indices and on Tauberian theorems on oscillation for (ϕ, λ) method would reveal. Moreover, he gave the precise answer that under the hypotheses in (α) the sequence $\{s_n\}$ of partial sums $s_n = a_0 + a_1 + \ldots + a_n$ has upper limit s. Later, he showed that the Tauberian condition on the left of (α) can be weakened to a slow-decrease type of condition, viz.,

$$\lim_{n \to \infty}\ \inf_{\lambda_n < \lambda_m < (1+\epsilon)\lambda_n} (a_{n+1} + \ldots + a_m) = O_L\,(1) \text{ as } \epsilon \to 0$$

to get the same answer. Again he showed that under the hypotheses on the left of (α) the sequence $\{s_n\}$ converges in λ-density to s in the following sense : A sequence of positive integers is said to have λ-density $\triangle_\lambda$ provided

$$\lim_{x \to \infty} \frac{1}{x} \int_0^x S_\nu^*\,(t)\, dt \equiv \triangle_\lambda \text{ exists, where } S_\nu^*\,(t) = 0,\ 0 \leqslant t < \lambda,\ s_\nu^*\,(t) = 1 \text{ or } 0$$

according as ν is or is not in the sequence when $\lambda_\nu \leqslant t < \lambda_{\nu+1}$. A real sequence $\{s_n\}$ is said to converge to s in λ-density if it converges to s through a sequence of positive integers of unit λ-density.

CTR in his work on 'convergence in density' (1957) exploited the notion to obtain an analogue of the classical Wiendr theorem with an assertion of the coincidence of the oscillation limits of two summability transforms as the conlcusion. The culmination of the work inspired by that of his revered teacher, mentioned earlier, found expression in a memorial volume devoted to his teacher K. Ananda Rau, in 1969, edited chiefly by him. In his paper therein, CTR gave a direct proof of the extended form of his result for summability (Φ, λ). The (Φ, λ) transform is

$$(\gamma) \qquad \Phi_\lambda(t) = \sum_{n=1}^{\infty} a_n \, \Phi\,(t\lambda_n),\ t > 0,$$

and we are concerned with $t \to +0$ $\sum_{n=1}^{\infty} a_n$ is said to be summable (Φ, λ) to s when $\Phi_\lambda\,(t)$ (assumed to exist for $t > 0$) tends to s as $t \to 0$. For the choice $\Phi(u) = (1-u)^k$, $k > 0$, $0 \leqslant u \leqslant 1$, $\Phi(u) = 0$, $u > 1$, (γ) defines the (R, λ, k) transform. In the following general result of Rajagopal, the proof of which utilizes a generalization to summability (R, λ, k) of his result mentioned just above for summability $(R, \lambda, 1)$. Rajagopal has given a conclusive result which is true, of not only the Rieszian transform but also of a general class of transformations which includes the generalized Abel transforms (A_α, λ), $\alpha > -1$, the Riemann transform $(R, \lambda, 2)$ the Stieltjes transform (S, ρ, λ), $\rho > 0$ and the Lambert transform (L, λ).

(I) Let the transform $\Phi_\lambda(t)$ be defined as in (γ) with $\Phi\,(u) = \int_u^\infty \psi\,(x)\,dx$ and $\psi\,(u)$ satisfying the conditions : $\psi\,(u) \geqslant 0 (u>0)$, $\int_0^\infty \psi\,(u)\,du = 1$, $\int_0^\infty \psi\,(u)\,|\log u|\,du < \infty$, $\int_0^\infty u^{ix}\,\psi\,(u)\,du \neq 0\ (-\infty < x < \infty)$.

Then the condition

$$(\delta) \quad \begin{cases} \lim\limits_{\varepsilon \to 0}\ \lim\limits_{n \to \infty}\ \min\limits_{\lambda_n > \lambda_m < (1+\epsilon)\lambda_n} (a_{n+1} + a_{n+2} + \ldots + a_m) \geqslant 0, \\ \overline{\lim\limits_{n\to\infty}}\ (\,|\,a_n\,| - a_n)\,/2 = h < \infty \end{cases}$$

lead to the conclusion

$$\lim_{t \to 0} \Phi_\lambda\,(t) = s \Rightarrow \overline{\lim_{s}}\ s_n = s - h$$

If (δ) is replaced by either of the two stronger conditions

$$(\delta'') \quad \begin{cases} \lim\limits_{\epsilon \to 0}\ \lim\limits_{n \to \infty}\ \min\limits_{\lambda_n \leqslant \lambda_m < 1 = \epsilon)\lambda_n} (a_{n+1} + a_{n+2} + \ldots + a_m) \geqslant 0 \\ \text{and } \lim a_n \geqslant 0 \text{ or } \lim \dfrac{\lambda_{n+1}}{\lambda_n} = 1 \end{cases}$$

$$(\delta'') \quad \lim_{\epsilon \to 0}\ \underline{\lim_{n \to \infty}}\ \min_{\lambda_n \leqslant \lambda_m < (1+\epsilon)\lambda_n} (a_n + a_{n+1} + \ldots + a_m) \geqslant 0$$

the conclusion will become :

$$\lim_{t \to 0} \Phi_\lambda(t) = s \Rightarrow \lim_{n \to \infty} s_n = s.$$

Besides these wide and deep generalizations, CTR obtained in his note (1946) on the oscillation of Riesz means of any order precise inequalities involving the Riesz means of orders k and $k+1$ under the hypothesis

$$a_n = O_L\left(\frac{\lambda_n - \lambda_{n-1}}{\lambda_n}\right), n \to \infty,$$

which is one of the hypotheses on the left of (α). These inequalities yield for $k = 0$ the earlier result (β).

Another interesting aspect of Rajagopal's work relating to (Φ, λ) summability is revealed by a sort of converse for high-indices theorems and which generalizes an earlier result of M. C. Austin [*J. London math. Soc.*, **26** (1951), 304-307] for (A, λ) summability.

(II) If the function (u) defining the transform $\Phi_\lambda(t)$ satisfies the conditions imposed on it in (I), and if only series with bounded terms are (Φ, λ) summable, then the highindices condition

$$\lim \frac{\lambda_{n+1}}{\lambda_n} > 1$$

is fulfilled and the series which are summable turn out to be convergent.

CTR's concern for obtaining the most general result in any particular direction is amply illustrated by his treatment of what are now familiarly known as Tauberian constants. To trace briefly the developments in this direction, it is convenient to recall first what Hardy [*Divergent Series*, Oxford, 1949, p. 306, Theorem 238] calls Vijayaraghavan's theorem.

(III) Let

$$\tau(x) = \sum_{n=0}^{\infty} C_n(x)\, s_n, \text{ where } C_n(x) \geqslant 0,\ C_n(x) \to 0,\ x \to \infty,\ \sum_{n=0}^{\infty} C_n(x) = 1,$$

be a totally regular sequence to function transform of the sequence $\{s_n\}$. Then

$$\tau(x) = O(1),\ x \to \infty \Rightarrow s_n = O(1),\ n \to \infty,$$

when

$$\lim_{t\to\infty}\ \inf_{0 < \Phi(u) - \Phi(t) < \eta} \{s(u) - s(t)\} \geqslant -w,\ 0 < w < \infty \text{ for some } \eta > 0,$$

where $s(t)$ is defined by

$$s(t) = s_n,\ n \leqslant t < n+1,\quad t \geqslant 0,$$

and $\Phi(x)$ is a strictly increasing unbounded function of $x > 0$ such that $\Phi(x+1) - \Phi(x) \to 0$, $x \to \infty$, with

$$\sum_{n=0}^{M} C_n(x) \to 0 \text{ if } M \to \infty,\ x \to \infty,\ \Phi(x) - \Phi(M) \to \infty,$$

$$\sum_{n=N}^{\infty} C_n(x)\,[\Phi(n) - \Phi(N)] \to 0 \text{ if } N \to \infty,\ x \to \infty,\ \Phi(N) - \Phi(x) \to \infty.$$

Summability of $\{s_n\}$ to l by the (τ) transform is the relation $\tau(x) \to l$, $x \to \infty$. The transformation $\tau(x)$ includes the Abel, Borel and Riemann-second-order transformation corresponding respectively to

$$C_n(x) = (1-e^{-1/x}\ e^{-n/x},\ C_n(x) = e^{-x}\ \frac{x^n}{n!},\ C_n(x) = \frac{2}{\pi x}\left\{\frac{\sin \frac{n}{x}}{\frac{n}{x}}\right\}^2.$$

Vijayaraghavan's theorem is a supplement to Wiener's general theorem being useful in deducing the Tauberian theorems with the Schmidt form of Tauberian condition, for instance, for the transformations mentioned just above. Vijayaraghavan's theorem can be interpreted as asserting that certain Tauberian conditions, which make the convergence of $\{s_n\}$ follow from that of $\tau(x)$, make also the boundedness of $\{s_n\}$ follow from that of $\tau(x)$. The class of results which are said to give Tauberian constants, on the other hand, relate the behaviour of $\{s_n\}$ as $n \to \infty$ to that of $\tau(x)$ as $x \to \infty$ through a sequence of values depending in cach case on the Tauberian condition selected. The first to investigate this aspect were H. Hadwiger [*Comment. Math. Helv.*, **20** (1947), 319--332] and R. P. Agnew [*Duke Math. J.* **12** (1945), 27–36], who treated the Abel transform. Rajagopal, in some of his generalizations independently of Vijayaraghavan, showed that in the case of the Borel transform $\tau(x) \equiv B(x)$

$$(\epsilon)\quad \lim_{n\to\infty} \mid B(n) - s_n \mid \leqslant \left(\frac{2}{\pi}\right)^{1/2} \lim_{n\to\infty} \mid n^{1/2}\,(s_n - s_{n-1}) \mid,$$

on the assumption that the right side is finite, the Tauberian constant $(2/\pi)^{1/2}$ being the best possible in the sence that it cannot be replaced by a smaller one. More than the fact that (ϵ) was obtained by Rajagopal, it is important to note that it is a class of summability methods which was treated in each of his papers on the Tauberian series and this class included one or more of the well-known transforms : Abel, Borel, Lambert, Riesz, Stieltjes. In fact, in the last of the series of papers on this particular topic in 1974, Rajagopal indicated how such results not only include results of original Tauber's type but also the Hardy-Littlewood-Wiener type of Tauberian theorems with two-sided Tauberian conditions in the case of a Karamata transform

$$\Psi(x) = \int_0^\infty \Psi(x, u)\, s(u)\, du$$

where $\Psi(x, u) \geqslant 0$ and as $x \to \infty$, $\int_0^U \Psi(x, u)\, du \to 0$ (for fixed U),

$$\int_0^\infty \Psi(x, n)\, du \to 1 \text{ as } x \to \infty.$$

Though R. P. Agnew [*Trans. Am. math. Soc.*, **72** (1952), 501–518] and H. Delange [*Ann. Sci. Ecole Norm Sup.*, (3) **67** (1950), 99–160] studied Tauberian constants, Rajagopal's investigations were extensive.

Another direction in which Vijayaraghavan and Rajagopal extended Vijayaraghavan's theorem is the following :—

If, in that theorem the main hypothesis $\tau(x) = 0(1)$ is changed to $\tau(x) = O_R(1)$ and the Tauberian condition altered to (π) $s_n - s_{n-1} \equiv a_n = O_L(1/g(n))$, $n \to \infty$, where

$g(x)$ is a function depending on the transform and tending steadily to ∞ with x, the conclusion gets altered to $s_n \leqslant C_\tau\ G\ (\log n) + 0(1), n \to \infty$, where $G(x) = \int^x dt/g\ (t) \to \infty$ and C_τ is a Tauberian constant for the summability method τ, the altered conclusion being the best possible in the sense that equality is realized in special cases for a sequence of values of n tending to ∞. Further, a Tauberian condition more restrictive than (π) ensure $\tau(x) \to -\infty \Rightarrow s_n \to -\infty$ as in an earlier result of Vijayaraghavan [*J. London math. Soc.*, **2** (1927), 215–222] for the Abel and Borel transformations. Rajagopal presented this collaborative work posthumously after his friend's demise in 1955 and 1956.

The utility of what are now called product theorems (involving the composition of two summability methods) is well known in Tauberian theory. The earliest utilization of such a theorem, perhaps, dates back [*see* G. H. Hardy, *Divergent Series*, p. 220, Theorem 156] to the Hardy-Littlewood proof of the O–Tauberian theorem for the Borel transform. O. Szasz [*Proc. Am. math. Soc.*, **3** (1952), 257–263) ; *Ann. Polon. Math.*, **25** (1952–53), 75–84] revived the interest in product theorems involving special methods of summability. CTR generalized Szasz's theorems so that the components of the product were either regular sequence-to-sequence or continuous Hausdorff transforms or else power series transforms of the form

$$J\,(x;\, s_n) = \frac{\sum\limits_{n=0}^{\infty} p_n x^n s_n}{\sum\limits_{n=0}^{\infty} p_n x^n}\quad,\ p_n \geqslant 0,\ 0 < x < \rho,$$

or else integral transforms of the form

$$(\eta)\qquad \Psi\,(t) = t \int_0^{\infty} \Psi(xt)\, s(x)\, dx,\ t > 0,$$

with desirable restrictions on Ψ or even of the form (reducing to (γ))

$$(\omega)\qquad \Psi\lambda(t) = \int_0^{\infty} \phi(xt)\ d[s(x)]$$

when $s(x)$ is the λ-step function

$$\begin{aligned} s(x) &= s_n,\ \lambda_n \leqslant x < \lambda_{n+1},\ n \geqslant 1,\\ &= 0,\ 0 \leqslant x < \lambda_1, \end{aligned}$$

with reasonable restriction on Φ. Most of the familiar transforms are covered by the choices made by him. By combining his product theorem with a 'positive' Tauberian theorem Rajagopal also obtained general Tauberian theorems for the transforms (η) and (ω) yielding well-known theorems in some special cases in 1952 particularly those of Szadśz. The work on product theorems led to collaboration with A. Jakimovski in 1954 providing the latter a starting point for his comprehensive work on the same theme and later with M. R. Parameswaran in 1960, who is currently engaged in exploiting product theorems to simplify the proof of several Tauberian theorems.

R. D. Lord [*Proc. London math. Soc.*, (2) **38** (1935), 241–256] first established a connexion between Borel summability and Cesáro summability of a sequence through a Tauberian condition. His result is : If the real sequence $\{s_n\}$ is summable (B) to s and the nth Cesáro mean of order α, $s_n = O_L(n^{k+1/2})$, $\alpha \geqslant -1$, $k \geqslant 0$, then s_n is summable $(C, \alpha + 2k + 1)$ to s. Rajagopal in his paper connecting Borel and Cesáro summabilities in 1960 showed that the Tauberian hypothesis in the result could be imposed on blocks of terms instead of on individual terms by proving that : if $\{s_n\}$ is a real sequence summable (B) to s and there is a $k \geqslant 0$ for which

$$\lim_{\delta \to +0} \quad \lim_{m \to \infty} \quad \min_{m \leqslant n \leqslant m + \delta\sqrt{m}} \quad \frac{s_n - s_m}{m^k} \geqslant 0,$$

then $\{s_n\}$ is summable $(C, 2k)$ to s. This inspired work of the same kind relating to other methods of the "Circle family" for instance the Meyer-Konig-Ramanujan method (S. α) by Y. Sitaraman *math. Z.*, **95** (1967), 34–49] and an unpublished work of V. K. Krishnan. covering these methods, for the $F(a, q)$ family introduced by A. Meir [*Ann. math.*, (2) **78** (1962), 594–599]. As a matter of fact in 1976. Rajagopal himself considered such a family $F^k(a, q,)$ of methods [in conjunction with G. Faulhaber's (*Math. Z*, **66** (1956), 34–52) $V(a,\lambda$ methods)] and obtained the following analogue of a classical result of Knop : If the nth Cesàro mean s_n^p of order $p > 0$ is such that $s_n^p = s + 0\,(n^{-p/2})$, then $s_n \to s$ $(F^k(a, q))$.

Rajagopal had also introduced, independently of Jakimovski and almost simultaneously, a method of summability parametrized by $\alpha > -1$. This method, in the case of a functicn defined for $x > 0$, was later [*Compositio math.*, **19** (1968), 167–195] called (L, α) method while its analogue for sequences was called (A_α) method and discussed by Borwein [*Q. J. math. Oxford Ser.*, (2) **9** (1958), 310–316]. Both these methods form scales of methods. Rubel [*Pacific J. math.*, **10** (1960), 997–1107] noticed an interesting feature of the scale of methods (L, α) for functions $s(x)$ bounded in $(0, \infty)$ and Rajagopal adapted this feature, with a simplified proof, for functions bounded on one side in $(0, \infty)$. He further enhanced the utility of the (A_α) method by directly exhibiting how the hypothesis of summability (A) $[(A_\alpha)$ for $\alpha = 0]$ in Tauber's original first and second theorems could be replaced by the weaker assumption of (A_α) summability, $\alpha > 0$. It is pertinent to remark that Rajagopal's conjecture that the weaker (A_α) summability, $\alpha > 0$, could perhaps replace the assumption of Abel summability in most of the Tauberian results has come true because of later work (*see* for instance *Tohoku math. J.*, **16** (1964), 257–269) and most recently in the case of the gap Tauberian theorems too as shown by V. K. Krishnan [*Math. Proc. Camb. phi. Soc.* **78** (1975), 497–500].

S. Minakshisundaram and CTR gave a set of difference formulae applicable to Riez typical methods (R, λ, k) of summability for series of order $k > 0$. These formulae, were obtained independently of L. S. Bosanquet's [*J London math. Soc.*, **18** (1943), 239–248] somewhat different set of formulae. Both these sets of formulae have been used by their authors for extending certain results of K. Ananda Rau [*Proc. London math. Soc.*, (2) **34** (1932), 414–440] involving the (R, λ, k) methods of order $0 < k \leqslant 1$ to $k > 1$. K. Chandrasekharan and S. Minakshisundaram [*Typical Means*, § 3.8] used Bosanquet's formulae to extend some interesting converse theorems on the abscissae of summability of general Dirichlet series. These extensions themselves were further extended in simpler as well as more meaningful results of Rajagopal. A typical result is

(IV) Suppose that (*i*) the Dirichlet series $\sum\limits_{n=1}^{\infty} \dfrac{a_n}{\lambda_n^s}$, $s = \sigma + i\tau$, is summable (R, λ_n, q) for some $q \geqslant 0$ when $\sigma > \rho$, (ii) the sum function $f(s)$ thus defined is regular for $\sigma > \eta$ when $\eta > \rho$ and satisfies the condition $f(s) = O(|\tau|^r)$, $r > 0$, uniformly for $\sigma \geqslant \eta + \epsilon > \eta$, (iii) the coefficients a_n of the Dirichlet series satisfy one of the two conditions

$$\text{(a)} \quad \overline{\lim_{n\to\infty}} \max_{\lambda_n \leqslant \lambda_m \leqslant \epsilon\,\theta(\lambda_n)} \frac{a_n + a_{n+1} + \ldots + a_m}{\lambda_n^\rho} = O_R(1).\ \epsilon \to 0,$$

or

$$\text{(b)} \quad \overline{\lim_{n\to\infty}} \max_{\lambda_n \leqslant \lambda_m < \epsilon\,\theta(\lambda_n)} \frac{a_{n+1} + a_{n+2} + \ldots + a_m}{\lambda_n^\rho} = O(1),\ \epsilon \to 0$$

where $\theta(x) = x^{1-(\rho-\eta)/r}$. Then the Dirichlet series is summable (R, λ_n, k), $0 \leqslant k < r$, for $\sigma \geqslant \{(r-k)\rho + k\eta\}/r$.

CTR concentrated his attention for some two years (1962–63) on the study of Fourier series. He first refined a Tauberian theorem for multiple Fourier series due to Chandrasekh ran and Minakshisundaram [*Typical Means*, p. 119, Theorem 4.53]. This refinement is incidentally a precise analogue of the best possible result of F. T. Wang [*Proc. London math. Soc.*, (2) **47** (1942), 308–325, Theorem 8] for a single Fourier series, but with a proof which is not quite analogous to that of the latter result. In his work on the Nörlund summability of Fourier series, he obtained a result at a point which includes two apparently different results one of them developed by Fejer, Lebesgue and Hardy [A. Zygmund, *Trigonometrical Series*, Chelsea, 1952, p. 49, § 3.31] and the other by K. S. K. Iyengar [*Proc. Indian Acad. Sci. Sect.* **A18** (1943), 81–87, Theorem II] and J. A. Siddiqi [*Proc. Indian Acad. Sci. Sect.* **A28** (1949), 527–531]. In another paper in 1963, he made a neat generalization of earlier results so as to include results of S. Izumi and G. Sunouchi [*Tohoku math. J.*, (2) **3** (1951), 298–305] and M. Sato [*Proc. Japan Acad.*, **30** (1954), 809–813] for uniform convergence of Fourier series at a point on the one hand and of results about ordinary convergence at a point due to Hardy and Littlewood *Ann. Scoula Norm. Sup. Pisa*, (2) **3** (1934), 43–62] and an extension thereof by W. Jurkat *Math. Z.*, **53** (1950/51), 309–339]. His result after desirable simplifications, without losing generality, is the following : Let $f(t)$ be even, of period 2π, $f(t) \in L(-\pi, \pi)$, $f(t) \sim \frac{1}{2}a_0 + \sum\limits_{m=1}^{\infty} a_m \cos mt$. Suppose that $f(x') - f(x'') = O\left\{\dfrac{1}{L(1/|x'-x''|)}\right\}$ as x', $x'' \to 0$, where $L(u) \to \infty$, $L(ku)/L(u) = O(1)$ for any $k > 0$ as $u \to \infty$ and further that, as $n \to \infty$,

$$\text{(i)} \quad \sum_{m=1}^{\infty} \frac{|a_m|}{|m-n|} = O\left(\frac{e^{CL(n)}}{n}\right) \text{ for some } C > 0,$$

$$\text{(ii)} \sum_{m=1}^{\infty}{}' \frac{|a_m|}{|m-n|} = O(1),$$

where the dash denotes the omission of the term in Σ corresponding to $m = n$. Then the Fourier series of $f(t)$ is "uniformly convergent" at $t = 0$. We can take for $L(u)$-log (u) as Izumi and Sunouchi (*loc. cit.*) did or (log log u^{α}), $\alpha > 1$, $(\log u)^{\alpha}$, $0 < \alpha < 1$, log log u with (i), (ii) replaced by

$$a_n = O\left(\frac{e^{CL(n)}}{n}\right) \text{ for some } C > 0.$$

Rajagopal's attention was especially directed to (i) gap Tauberian theorems, (ii) high-indices theorems and (iii) the history of medieval Kerala mathematics in the last two or three years before his passing away. To a great extent the interest in these topics shared with him by the author of this memoir has been responsible for this special attention. Both he and the author had been meeting in either his or the latter's residence at least twice a month spending several hours, and a good part, on discussing some problem or other in these topics. The inspiration for the study of topic (*i*) came from the author's [*Publ. Ramanujan Inst.* **1** (1969), 269–281] successful use of an idea of V. I Mel'nik's to unify the slow osciallation type of Tauberian theorems and the gap theorems for certain well known methods of summability such as the Abel, Borel and Lambert. CTR raised the question as to whether we had an analogue of Mel'nik's idea for λ-type methods and this was settled later in 1972. Explicitly, it was proved that if $\Phi_{\lambda}(t)$ is the transform corresponding to one of the typical methods, Abel, Lambert or Riesz, and if

$$A(u) = \sum_{\lambda_n < u} a_n$$

satisfies the condition

$$\lim_{n \to \infty} \sup_{\lambda_n < u < (1+\delta)\lambda_u} |A(u) - A(\lambda_n)| \to 0 \text{ as } \delta \to 0$$

then

$$\lim_{t \to \infty} \Phi_{\lambda}(t) = s \Rightarrow \lim_{u \to \infty} A(u) = s.$$

Further, that if

$$\lim_{n \to \infty} \sup_{\lambda_n < u < (1+\delta)\lambda_n} |A(u) - A(\lambda_n)| = 0 \text{ for every } \delta > 0,$$

$$\lim_{u \to \infty} A(u) = \lim_{t \to +0} \Phi_{\lambda}(t).$$

This result is new for the typical Lambert method though it includes the known high indices theorem for the method due to Levinson [Gap and density theorems, *Am. math. Soc. Colloq. Publ.*, No. 27, (1940)] and is interesting in the absence of any known implication of the form $(L, \lambda) \Rightarrow (A, \lambda)$.

It was Rajagopal who first noticed the connexion between the function Φ introduced by Hardy in the context of Vijayaraghavan's theorem (III *supra*) and gap

Tauberian theorems for the method (τ). In one of the Lemmas in 1969 in his article on 'gap Tauberian theorem's he proved the following general result :

(V) Let the transform $\tau(x)$ be defined as in (III) with an associated function Φ.

If the series $\sum_{n=0}^{\infty} a_n$, $a_n = s_n - s_{n-1}$, $n = 0, 1,2, \ldots,$ $s_{-1} = 0$, has gaps as follows :

$$a_n = 0 \text{ if } n \neq n_k,\ k = 0,1,2,\ldots,$$
$$\Phi(n_{k+1}) - \Phi(n_k) \to \infty, \quad k \to \infty,$$

then under the hypothesis $s_n = O_L(1)$, $n \to \infty$ we have the conclusion

$$\lim_{n \to \infty} s_n = \lim_{x \to \infty} \tau(x).$$

In the special case of the Borel transform $B(x)$, he further proved that under the more restrictive gap condition

$$a_n = 0 \text{ if } n \neq n_k,\ k = 0, 1, 2,\ldots,$$
$$n_{k+1} - n_k > \theta_{n_k}^{2/3} \quad (\theta \geqslant 0, \text{ fixed})$$

the less restrictive Tauberian condition $s_n = 0\,(\exp n^{\alpha})$ for some $\alpha < 1$, $n \to \infty$ leads to the conclusion

$$\lim_{n \to \infty} s_n = \lim_{x \to \infty} B(x).$$

The collaborative work of Rajagopal and the author on gap Tauberian results continued almost till the end of the former's life and, in fact, there is a well-prepared MS by both awaiting publication after incorporation of certain valuable contributions of Professor B. Kuttner and making him one of the authors as very much desired by Rajagopal and the author too. It is shown therein that when a transform $\tau(x)$ defines a "Karamata method" of summability with Φ associated as in (III), either the slow decrease type of condition

$$\lim_{\eta \to 0} \lim_{n \to \infty} \inf_{0 \leqslant \Phi(n') - \Phi(n) \leqslant \eta} (s_{n'} - s_n) \geqslant 0$$

or the following gap condition strengthened by a boundedness assumption added to it

$$a_n = 0,\ n \neq n_k,\ \Phi(n_{k+1}) - \Phi(n_k) \geqslant C > 0 \ (C \text{ fixed})$$
$$\lim s_n > -\infty$$

ensure the implication

$$\tau(x) \to s,\ x \to \infty \Rightarrow s_n \to s,\ n \to \infty.$$

In continuatio of Hirokawa's [*Tohoku math J.*, (2) **7**, (1955), 279–295] work, Rajagopal studied a combination of Riemann and Cesàro methods called the Riemann-Cesàro methods of summability. A λ-type analogue of this concept and another involving the Bessel function was introduced and studied by Rangachari [*Math. Z.*, **88** (1965), 166–183 and **91** (1966), 344–347]. Summability (R, λ, p) of

$\sum_{n=0}^{\infty} a_n$ to s means that $\sum_{n=1}^{\infty} a_n \left(\frac{\sin\lambda_n^{t}}{\lambda_n^{t}}\right)^p$ converges for $0 < t < t_0$ and tends to s as $t \to +0$. Here $0 < \lambda_n \uparrow \infty$. When $\lambda_n = n$, B. Kuttner [*Proc. London math. Soc.*, (2) **38** (1935), 273–283] showed that $(\mathring{R}, \lambda_n, 2)$ summability of $\sum_{n=0}^{\infty} a_n$ to s implies its Cesàro summability $(C, 2 + \delta), \delta > 0$, to s and hence its Abel summability to the same sum. For λ-type methods Rajagopal and Rangachari showed in 1972 that if $\sum_{n=0}^{\infty} a_n$ is $(\mathring{R}, \lambda, 2)$ summable to s and $\sum_{r=0}^{\infty} (\mid a_r \mid - a_r)\, \lambda_r = O(\lambda_n)\ n \to \infty$, then $\sum_{n=0}^{\infty} a_n$ is (A, λ) summable to s. Kuttner [*Proc. Camb. phil. Soc.*, **75** (1974), 83–94], however, showed that the Tauberian condition was superfluous, by adapting certain techniques of Rajchman and Zygmund [*Math. Z.*, **24** (1926), 47–104] but there are other non-trivial results and among them the one giving the condition

$$\lim_{\eta \to 0} \quad \lim_{n \to \infty} \quad \sup_{\lambda n \leqslant (\lambda_r < (1 + \eta)\lambda n} \quad \sum_{(r)} (\mid a_r \mid - a_r) = 0$$

as a Tauberian conditions for $(\mathring{R}, \lambda, 2)$ summbility of $\sum_{n=0}^{\infty} a_n$ to imply its convergence. This result yields the pure high indices theorem for $(\mathring{R}. \lambda, 2)$ summability without the sophistication involved in the later work of Kuttner's.

*Functions of a Complex Variable**

Rajagopal's early results (*see* e. g., *a* paper published in 1945) were on periodic meromorphic functions and were inspired by Ganapathy Iyer's work [*J. Indian math. Soc., N. S.*, **5** (1941), 1–17]. He showed in another paper in 1947 that if f (z) is a periodic integral function of order $\rho > 1$ and

$f\ (z) = a_0 + \sum_{n=1}^{\infty} 2\ \{a_n \ \cos\ (2\ \pi nz/\lambda) \pm b_n\ \sin\ (2\ \pi nz/\lambda)\}$, then the Fourier coefficients a_n, b_n satisfy the condition $a_n, b_n = O$ (exp $(-n^{\rho^*} - \epsilon)$ for all positive ϵ, ρ^* being defined by $\frac{1}{\rho} + \frac{1}{\rho^*} = 1$. He also obtained an analogue of Landau's theorem [*Darstellung und Begrundung einiger neurer Ergebnisse der Funktionentheorie*, Berlin, 1929, §§ 1, 2, 4] on the partial sums of Fourier series and gave a new proof of Carathéodory's inequality and allied results [*see* D. S. Mitrinovic, *Analytic Inequalities*, Springer, 1970, p. 333, 3. 8. 41]. The analogue [*see* note on Poores series 1952] of Landau's theorem mentioned above is that for $f\,(z) = \sum_{n=0}^{\infty} a_n z^n$ analytic in

*The author owes the review relating to Rajagopal's work on function theory to Professor T. V. Lakshminarasimhan, former student and collaborator of his. The biographical part of this memoir elaborates a notice by Professor Lakshminarasimhan and the author published in *The Hindu* dated June 20, 1978. Most of the details about Rajagopal's family were kindly supplied by Mrs Rukmini Rajagopal.

$| z | \leqslant R$ we have $| \text{Re}\, s_n (z) | < C \log$ -$nA)^*(r)$ wheie $s_n(z) = \sum_{k=1}^{n} a_k z^k$ and $A^*(r)$

$= \max | \text{Re} f(z) |$ for $| z | = R$. Using this and Wiman-Valiron technique he obtained (as given in his article on an asymptotic relation between an entire function, its deviation and order, an analogue of a relation, originally given in part by G. Pólya and G. Szegö (*Aufgaben und Lehrsatze aus der Analysis*, Berlin, 1925) and completely by S. M. Shah [*Bull Am. math. Soc.*, **53** (1947), 1156–1163], connecting the growth of the maximum modulus of an entire function $f(z)$, its derivative $f'(z)$ and their order ρ and lower order λ, namely

$$(\xi) \quad \overline{\lim_{r \to \infty}} \log r \{M(r, f')/M(r, f)\} \log r = \lambda^{\rho} \ (0 \leqslant \lambda, \rho \leqslant \infty).$$

He also proved that for an entire function $f(z)$ of order ρ, $0 \leqslant \rho \leqslant \infty$,

$$\lim_{r \to \infty} \frac{(\max | \text{Re}\, zf'(z) | / \max\limits_{| z | = r} | \text{Re} f(z) |}{\log r} = \rho.$$

The corresponding result with $\overline{\lim}$ replaced by lim and the order ρ by the lower order λ of $f(z)$ is still an open question. Motivated by an advanced problem of H. S. Shapiro [*Am. math. Mon.*, **59** (1952), 45] he also obtained in 1953 the inequality (*see* R. P. Beas, Jr., *Entire Functions*, Academic Press, 1954, p. 7, footnote 1.3a.)

$$f'(z) | \leqslant \frac{2R}{R^2 - r^2} \quad \frac{M(R)}{e}, 0 \leqslant | z | = r < R,$$

for the function $f(z)$ regular and non-vanishing in $| z | < R$. In the theory of entire functions, he obtained in 1941 a new proof of Hadmard's factorization theorem and collaborated with Lakshminarasimhan and others in supplementing several results obtained by Lakshminarashimhan, in an attempt by the latter, to investigate the analogues to (ξ) that can be obtained by replacing in (ξ) $M(r, f)$ and $M(r, f')$ by other growth functions associated with an entire function $f(z)$ and its derivative $f'(z)$. It was shown by him and Lakshminarasimhan later in 1970 that if (*A*) (*i*) log $S(r)$ is a non-negative steadily increasing convex function of $\log r$ for $r > r_0 > 0$;

(ii) $\log S(r)$ is of finite non-zero order ρ and finite type τ; then $\overline{\lim\limits_{r \to \infty}} \ S_1(r)/S(r)$ $r^{-1} \leqslant e^2 \rho\, \tau$, where e^2 is the best possible constant.

(B) If we suppose additionally that (iv) there is a constant $O \geqslant 0$ such that

$$S_1(r) \geqslant S'(r) - r^{-1} CS(r)$$

whenever $S'(r)$ exists and is finite, that is, except possibly for a countable set r where $S'(r)$ is to be replaced by $S'_{\pm}(r)$ the right and left derivatives of $S(r)$ then

$$\overline{\lim_{\eta \to \infty}} \quad S_1(r) / (S(r)\, r^{\rho-1}) \geqslant \rho\tau.$$

In the above, one can in particular make the choice $S_1(r) = M(r, f')$. $S'(r) = M(r, f')$, ρ, τ being the order and type of the entire function $f(z)$ and its derivative $f'(z)$. Of course, in this case and some other cases the growth functions $M(r, f)$ and $M(r, f')$ can be replaced by other growth functions such as $I_\delta(r, f)$ and $I_\delta(r, f')$ respectively, where

$$I_\delta(r, f) = \frac{1}{2\pi} \int_0^{2\pi} | f(re^{i\theta}) |^{\delta} d\theta)^{1/\delta} (0 < \delta < \infty).$$

While e^2 in the inequality in (ii) of (A) can be replaced by the smaller constant e by a different method and with particular growth function. Rajagopal was interested in similar results connected with the growth functions $M(\sigma, f)$, $M(\sigma, f)$ of an entire function $f(s)$ represented by an absolutely convergent Dirichlet series given by

$$f(s) = \sum_{n=1}^{\infty} a_n e^{\lambda_n s} \text{ with } s = \sigma + it,$$

$$\lambda_{n+1} \geqslant \lambda_n > 0, \lambda_n \to \infty \text{ as } n \to \infty \text{ with}$$

$$\text{(A) either } \overline{\lim_{n \to \infty}} \frac{\log n}{\lambda_n} = D < \infty$$

$$\text{(B) or } \overline{\lim_{n \to \infty}} \frac{\log n}{\lambda_n} = \epsilon = 0$$

and

$$M(\sigma, f) = \sup_{-\infty < t < \infty} | f(\sigma \pm it) | .$$

In collaboration with A. R. Reddy, he showed in 1965, for instance,

that if $\rho = \overline{\lim_{\sigma \to \infty}} \dfrac{\log \log M(\sigma)}{\sigma}$ and $\lambda = \underline{\lim}_{\sigma \to \infty} \dfrac{\log \log M(\sigma)}{\sigma}$, and

$$\mu(\sigma) = \max_{n \geqslant 1} | a_n e^{(\sigma + it)\lambda_n} \quad \text{and}$$

$$\rho_* = \overline{\lim_{\sigma \to \infty}} \frac{\log \log \mu(\sigma)}{\sigma}, \lambda_* = \underline{\lim}_{\sigma \to \infty} \frac{\log \log \mu(\sigma)}{\sigma},$$

then under condition (A)

$$\text{(i) } \rho = \rho_*, \ \lambda = \lambda_*,$$

$$\text{(ii) } \overline{\lim_{\sigma \to \infty}} \frac{\log \left[\dfrac{\mu'(\sigma)}{\mu(\sigma)} \right]}{\sigma} = \rho \text{ wherein}$$

$$\mu'(\sigma) \text{ refers to } f'(s) = \sum_{1}^{\infty} a_n \lambda_n e^{\lambda_n s}.$$

He established with Reddy again that historically and, of course, even otherwise important functional equation

$$(C)\ g(z+w)-\lambda g(z)=f(z),$$

where $f(z)$ is an entire function of order ρ $(0<\rho<\infty)$ and type σ $(0\leqslant\sigma<\infty)$, with w, λ non-zero constants, real or complex, had a solution with $g(z)$ entire, of order ρ and type λ. This was by way of setting right V. Krishnamurthy's [*Proc. natn. Inst. Sci. India,* Part A, **26** (1960), 642–655] defective proof of the same result by functional analytic methods to which Ganapathy Iyer drew his attention. After about two years, he added an addendum on a suggestion by Professor H. S. Shapiro which was in the nature of a self contained proof of his solution of the functional equation (C).

History of Medieval Kerala Mathematics

As stated earlier, taking the clue from an article by C. M. Whish [*Trans. R. asiat. Soc. Great Br., Ireland* **3** (1935), 509–523] Rajagopal, with the help of competent collaborators presented in two papers first in 1944 and later in 1949 (for the first time) satisfactory modern forms of the proofs of the three series named below, carefully moulding the proof, upon their originals in *Yuktibhasa,* a work in sanskritised form of Malayälam.

(*a*) Gregory's power series for arc tan x (1671 A.D.),

(*b*) Newton's power series for sin x, cos x (1670 A.D.).

A good summary of this work is given in the following two books on history of mathematics : J. E. Hofmann, *Geschichte der Mathematik,* 2nd edn., Berlin, 1963, **I**; and A. P. Juschkewitsch, *Geschichte der Mathematik in Mittelalter,* Leipzig 1964. The latter book contains an "unusually" (as with western work on history) thorough and up-to-date account relating to Near and Far East. Western prejudice to claims of contributions of the Orient goes even to the extent of blacking out the claims. In this context to contrast with B. Boyer's [*A History of Mathematics,* Wiley,(1968), 246–47] passing mention of "one of the reminders that mathematics owes to India," "the trigonometry of the sine function," and his mere listing of Hindu mathematics in a bibliography, it is worth recording what the Cambridge mathematical historian D. T. Whiteside has written in his monumental and valuable edition of the *Mathematical Papers of Isaac Newton,* **II,** Cambridge 1968, p. 237 Footnote (112) : These series for the sine and cosine, which here appear for the first time in a European manuscript, had, as we now know, already been displayed in a 1639 Malayalam compendium, the *Yuktibhasa,* which itself professes to be based on anearly sixteenth-century Sanskrit original, the *Tantrasangraha,* perhaps the work of the little-known Hindu mathematician Nilakantha (*see* A. R. A. Iyer's and R. Tampuran's modern Malayalam edition of the first part of the *Yuktibhasa* (Trichur, 1948) : 204 f.; and its review by C. T. Rajagopal and A. Venkatraman in [*Math. Rev.,* **12** (1951)], 309–310] The latter manuscript is still unpublished]. The Hindu approach, which depends for example, on repeated iteration of the identity.

$$.\sin x=\int[1-\int\sin x dx]dx,$$

is both wholly distinct from Newton's and was described in Europe only in the early nineteenth century (C. M. Whish,* 'On the Hindu Quardrature of the circle and the Infinite series of the proportion of the circumference to the diameter exhibited in the four sastras...,' *Trans. R. asiat. Soc. Gr. Br., Ireland,* **3** (1835) : 509–523), so that there can be no question of any influence upon Newton in his independent rediscovery of the series.

The astounding facts are, according to *prima facie* evidence and the latest bibliographic research : (i) *Yuktibhasa* is by an author whose period is 1500–1600 A.D., (ii) the series of Gregory and Newton were first formulated by Madhava, a Kerala Brahmin whose period is *c.* 1350–1400 A.D. A first step towards the establishment of (ii) was the article by himself and Rangachari written in 1878 which turned out to be the last published work of his. He was advised by the Cambridge mathematical historian Dr D. T. Whiteside, to whom this work was submitted before publication, to prepare a monograph updating his findings. He, however, preferred to complete this monograph in the form of research papers. With this plan, he and the author prepared another article highlighting the contents of a palm leaf MS. which they noticed recently in the Adyar Library and which agreed in all respects with Whish's account (*loc. cit.*). This was again submitted to Dr Whiteside whose valuable and frank report was received only on the day of Rajagopal's passing away. This article needs to be improved mathematically, following the suggestions of Dr Whiteside, and to be published.

HONOURS

Rajagopal was a life member of the Indian Mathematical Society and a member of the London Mathematical Society. He was an elected Fellow of the Indian Academy of Sciences (Bangalore) and of the Indian National Science Academy. He was President of the Allahabad Mathematical Society founded by his good friend,, the late B. N. Prasad. Rajagopal served the Indian Mathematical Society as its honorary Librarian for over 15 years. He, however, declined to be President for even one term, though persuaded by his friends in the Council of the Society. On the other hand, he presided over the mathematical section of the Indian Science Congress in 1963. His presidential address remains a carefully drawn and valuable survey of work in summability methods in India up till then.

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*Ibid., **III,** 1969, p. 34, Footnote (5) : ... *We may forgive him for not knowing that a century and half before, the Hindu mathematician Nilakantha had derived the inverse-tangent series by a 'reductio per divisionem', exactly as James Gregory and Leibniz were to do in January* 1671 *and late* 1673 *respectively : no hint of this reached Europe, it would appear, till* 1835, (*see* A. R. A. Iyer's and R. Tampuran's edition of the 1639 Malayalam compendium *Yuktibhasa,* Trichur, 1948 : 113 ff.; also C. T. Rajagopal and T. V. V. Aiyer, 'On the Hindu proof of Gregory's Series' and 'A Hindu Approximation toπ *Scripta Math.,* **17,** 1951 : 65–74; **18,** 1952 : 25–30; and K. M. Marar and C. T. Rajagopal, 'On the Hindu Quadrature of the Circle, *J. Bombay Branch R. asiat. Soc.,***20,** 1944 : 65–82). Both Gregory and Leibniz, of course, drew their inspiration from Mercator's quadrature of the hyperbola (*see* H. W. Turnbull's *Gregory Tercentenary Memorial Volume,* London, 1939 : 173, 537; and J. E. Hofmann's *Entwicklungsgeschichte der Leibnizschen Mathematik wahrend des Aufenthaltes* in Paris (1672–1676), Munich, 1949 : 32–36).

Personal Attributes

Rajagopal was a teacher par excellence at any level, graduate, post-graduate or advanced. His teaching was marked by very detailed treatment of the topic, be it either real or complex analysis or statics or dynamics; it did not only adhere to the main text prescribed for the topic but covered additional ground throwing light on the teacher's original study of the topic. Thus he catered very well to the average students while inspiring the bright ones towards original work. Among the students of the latter class should be mentioned : P. Kesava Menon, M. Parthasarathy, T. V. Lakshminarasimhan. As with his teacher, his way with research workers was to encourage and expect them to formulate their own problems and then to discuss the problems wih them.

The author cannot forget the day when he was advised by Rajagopal to work on summability methods revising his own earlier suggestion to work in the theory of groups. It was the day when the author could generalize Rajagopal's results relating to Nörlund summability. On the other hand, he was a very good collaborator who never distinguished his own contribution in a joint work from that of the others. Quite a few who were dedicated workers in other fields and who lacked support in their initial career were encouraged by CTR. One such, Professor K. Ramachandra, while unveiling the portrait of CTR in the Ramanujan Institute could not suppress his emotion when he remembered the encouragement given to him by CTR.

In the present day world of changing fashions in work in mathematics, Rajagopal never swerved from the principle of his revered teacher Ananda Rau, viz. to work in themes dear to us without caring for fads and fashions and to leave the assessment of the work to posterity. In this context, it is appropriate to quote from his Presidential Address to the Mathematics section of the Indian Science Congress, 1963 : As it has been said of old :

"There be of them, that have left a name
to declare their praises, behind them,
And some there be, which had no memorial,
Who are perished, as though they had not been;

but the fact of their having toiled in good faith shall remain continually "a good inheritance" to such as "follow in their foot steps..". Whenever we heard of mathematics in India it always meant the work of one or two, work certified to be superexcellent if not a creation of genius. We should be perfectly certain that mathematics in the country as a whole was a decaying activity. But if we hear of the patient efforts of a host of humble workers to keep alive the spirit of inquiry in mathematics, at different levels and in different directions, persistent efforts in the face of difficulties and discouragements, then we know that the country still is mathematically active. Rajagopal wrote well; whether it was his mathematical papers, or his articles on christianity or psychic studies or reviews or notices, he was very careful in the choice of idiom and of precise expression. Each one of the many letters he wrote to the author is a piece of literature. With no reservations he used to express his admiration for the work and ideas of his compatriots like B. N. Prasad, V. Ganapathy Iyer, S. Minakshisundaram, V. Ramaswami.

Rajagopal was a believer in rational thinking. He did not take interest in religious rituals, but at the same time did not discourage others believing in them. During India's freedom struggle he demonstrated his nationalism by always wearing Khadi in response to the call of Mahatma Gandhi. About philosophy, unlike his friend Vijayaraghavan, he felt that every one should have his own philosophy. Vijayaraghavan's personal discussions with him about *Visishtadvaita* Philosophy and the Tamil songs of the Vaishnavite saints did not impress him. He more recently used to tell the author that he wished Vijayaraghavan were alive to discuss these matters with the author. On the other hand, Rajagopal believed in the scientific investigation of the so called supernormal and superhuman phenomena. He was quite proud of his views in this aspect, viz., what goes by the name of psychic research, and more particularly in the rebirth of the soul, and was happy that Srinivasa Ramanujan and Ananda Rau too had the same belief. This is more than evident from what he writes in his notice on Ananda Rau. The author vividly remembers Rajagopal quoting from a western work and the former immediately pointing out that, his quotation is a free translation of the sloka

यं यं वापि स्मरन्भावं त्यजत्यंते कलेबरं।
तं तमेवैति कौंतेय सदा तद्राव भावित: ॥

[8–6]

from the *Bhagavadgita*. As one who had immense faith in the knowledge of the Indian seers and saints, Rajagopal remarked that it was but natural that these ideas were found in our own scriptures. Rajagopal had a valuable collection of literature pertaining to psychic research. His lighter reading material was high class detective stories.

Rajagopal was an exemplary gentleman. Nobody, knowing him closely has ever seen him raising his voice. If he ever became angry, it was always in defence of his colleagues and students. The Rajagopals were highly hospitable. The author and some of his colleagues cannot forget the long hours spent at their home. There was always a stream of visitors to their house in Tambaram where each one of the visitors was personally welcomed and treated by Rajagopal and his gracious wife Rukmini. He is survived by his wife and their only son C. R. Srinivasan for whom he showed lot of concern during his last days.

EPILOGUE

In the interest of scientists and scholars of all times, the author is constrained to end this notice with a saddening note. It is true that India did not offer facilities or congenial companionship to Srinivasa Ramanujan who found them only in England. But having known this truth, authorities in India do not seem to have any intention to learn a lesson from the folly of their predecessors. The authorities of the University of Madras ruled soon after Rajagopal's retirement that he was not eligible for any pension. He was not considered for an Emeritus Professorship either. He had thus to work on research projects to get financial assistance after retirement. With the intervention of eminent scientists like S. Chandrasekhar and A. Weil (*see* the notice by them in *Nature*, **279,** No. 5711, p. 358 May 24, 1979) an ex-gratia allowance was given to him for a few months till his death. His wife, however, was denied this allowance thereafter, even partially, even after she requested help. The blind and inflexible rule became handy for the authorities to deny her the help. Again, it is the same two eminent scientists, who are obviously high humanists too, who show concern to such souls. The author wishes that the governmental machinery in India and the scientific community in the country shows interest in such matters. It is hightime that a scheme or relief fund is instituted by the government or the Science Academy to help scientists or their families after retirement or death. The author wishes to conclude with fervent hopes that there would be a favourable response to this call.

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